Bergen County Academies Math Competition (Grade 7)
October 26th, 2003

1. A number plus 4 is 2003. What is the number?

2. Jane has 4 pears, 5 bananas, 3 lemons, 1 orange, and 6 apples. If she uses one of each fruit to make a fruit smoothie, what is the total number of fruits that she has left?

3. Compute $\frac{55}{21} \times \frac{28}{5} \times \frac{3}{2}$.

4. Philip has 3 triangles and 6 pentagons. Let S be the total number of sides of the shapes he has. Let N be the number of shapes he has. What is $S + N$?

5. On an exam with 80 problems, Roger solved 68 of them. What percentage of the problems did he solve?

6. Yao Ming is 7 feet 5 inches tall. His basketball hoop is 10 feet from the ground. Given that there are 12 inches in a foot, how many inches must Yao jump to touch the hoop with his head?

7. In triangle ABC, $\overline{BC} = 4$ and $\overline{CA} = 6$. If the perimeter of the triangle is 4 times the length of side $\overline{BC}$, what is the length of $\overline{AB}$?

8. Let A be the sum of seven 7's. Let B be the sum of seven A's. What is B?

9. Compute $664.02 \div 9.3$.

10. If a certain number is doubled and the result is increased by 11, the final number is 23. What is the original number?

11. Compute the product of the integers from -5 to 5, inclusive.

12. A problem author for a math competition was looking through a tentative exam when he realized that he could not use one of his proposed problems. Frustrated, he decided to take a nap instead, and slept from 10:47AM to 7:32PM. For how many minutes did he sleep?

13. In rectangle $ABCD$, $\overline{AB} = 7$ and $\overline{AC} = 25$. What is its area?

14. Evaluate $\frac{100-99+98-97+\cdots+4-3+2-1}{1-2+3-4+\cdots+97-98+99-100}$.

15. What is the area of a square in square feet, if each of its diagonals is 4 feet long?

16. A lazy student used the approximation $\pi = \frac{22}{7}$ to calculate the circumference of a given circle. If his answer was 6, what was the radius of the circle?

17. Find the largest divisor of 2800 that is a perfect square.

18. How many multiples of 17 are there between 23 and 227?

19. Two angles are supplementary, and one angle is 9 times as large as the other. What is the number of degrees in the measure of the larger angle?

20. How many positive whole numbers less than 100 are divisible by 3, but not by 2?

21. The surface area and the volume of a cube are numerically equal. Find the cube's volume.

22. Given that $|3 - a| = 2$, compute the sum of all possible values of a.

23. Let $ABCD$ be a square with side length 8. A second square $A_1B_1C_1D_1$ is formed by joining the midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$. A third square $A_2B_2C_2D_2$ is formed in the same way from $A_1B_1C_1D_1$, and a fourth square $A_3B_3C_3D_3$ from $A_2B_2C_2D_2$. Find the sum of the areas of these four squares.

24. If $a + b = 13$, $b + c = 14$, $c + a = 15$, find the value of c.

25. Two positive whole numbers differ by 3. The sum of their squares is 117. Find the larger of the two numbers.

26. Given that $5^3 + 5^3 + 5^3 + 5^3 + 5^3 = 5^J$ and $3^2 + 3^2 + 3^2 = 3^N$, what is the value of J^N?

27. A pair of positive integers a and b is such that their greatest common divisor is 5 and their least common multiple is 55. Find the smallest possible value of $a + b$.

28. How many of the positive divisors of 120 are divisible by 4?

29. How many three-digit numbers are perfect squares?

30. Calculate $1 + 3 + 5 + \cdots + 195 + 197 + 199$.

31. The ages of Mr. and Mrs. Fibonacci are both two-digit numbers. If Mr. Fibonacci's age can be formed by reversing the digits of Mrs. Fibonacci's age, find the smallest possible positive difference between their ages.

32. Let N be the product of the first nine multiples of 19 (i.e. $N = 19 \times 38 \times 57 \times \cdots \times 152 \times 171$). What is the last digit of N?

33. If two spheres have radii of 2 and 6, what is the ratio of the volume of the larger sphere to the volume of the smaller sphere?

34. If a, b, and c are nonzero numbers satisfying $3a = 4b$ and $5b = 6c$, what is $\frac{c}{a+b}$?

35. Point X is 210 miles from point Y. Car A starts at X and drives towards Y at 40 mph. Car B starts at Y and drives towards X at 50 mph. If both cars start at noon, at what time will they meet?

36. A slice of pizza costs \$1. A pie, which is composed of 8 slices, costs \$6. Bob buys individual slices and pies so that he has 78 total slices. If he has to pay \$62, how many slices does he buy individually?

37. If $A + B = \sqrt{8}$ and $A - B = \sqrt{5}$, what is $A \times B$?

38. Let $ABCD$ be a rectangle with $\overline{AB} = 20$ and $\overline{BC} = 6$. Let P be the point such that P is 12 units away from $\overline{DA}$ and 5 units away from $\overline{CD}$. What is the area of quadrilateral $APCD$?

39. What is the 100th digit after the decimal point when $\frac{1}{7}$ is written in decimal form?

40. Simplify $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \frac{1}{4\cdot 5} + \frac{1}{5\cdot 6} + \frac{1}{6\cdot 7}$ in lowest terms.

41. A circle has a diameter of $376,216$ inches. Using the approximation $\pi = 3.14$, compute the ratio of the circle's area, in square inches, to its circumference, in inches.

42. In isosceles triangle ABC, $\overline{AB} = \overline{AC}$ and $\angle BAC = 140^\circ$. Point D lies on $\overline{BC}$ such that $\overline{AD} = \overline{BD}$. Compute $\angle DAC$ in degrees.

43. What are the last 2 digits of 5^{2003}?

44. Which of the following are true, given that there may be more than one true statement:
I: the square of an integer is a whole number
II: if $a \neq b$ and $b \neq c$, then $a \neq c$
III: every integer has a rational inverse in multiplication
IV: the square root of a positive integer is real

45. A number is *strictly decreasing* if each digit is strictly less than the digit to its left. For example, 531 and 962 are *strictly decreasing*, whereas 562 and 322 are not. How many integers between 100 and 600 are *strictly decreasing*?

46. What is the units digit of $13^{17} + 17^{13}$?

47. There are ten lottery tickets in a hat, and four of them are winning tickets. First, Joe reaches in and takes a ticket. Then, Kim reaches in and takes a ticket from the remaining nine. What is the probability that Kim takes a winning ticket?

48. A *silly* number *ababab* is formed by repeating a two-digit number *ab* exactly three times. For example, 252525 is a *silly* number. What is the greatest common divisor of all *silly* numbers?

49. A number p yields a remainder of 3 when divided by 5, a remainder of 5 when divided by 7, and a remainder of 11 when divided by 13. If p is less than 1000, what is the maximum value of p?

50. In the *magic square* shown, numbers are to be placed in the empty boxes so that the sums of the numbers in each row, column, and diagonal are equal to the same value. What is the value of x?

19		
		26
	x	