1. Jim wants to get at least 90% of math competition problems correct. However, he has only answered 20 out of 50 correctly so far this year. What is the least number of problems he has to solve in the future in order to reach 90%?
2. Manchester United has been struggling, being outscored 20 to 12 over their first eight games. Given that their opponents never scored more than three goals in one game, what is the maximum number of wins Manchester United can have so far?
3. A right triangle ABC has right angle C and side AC=7, BC=24. What is the numerical difference between the area and the perimeter of the triangle?
4. Harry wants to order the distinct constants γ, e, π in increasing order. He knows that $\gamma < \pi$, then what is the probability that Harry correctly orders the 3?
5. Mr. and Mrs. Smith take a walk (at 6 miles per hour) from Hackensack to New York (a straight line distance of 12 miles). Christine decides to start from Hackensack and run to New York, turn back and meet Mr. and Mrs. Smith, run towards New York and so on until Mr. and Mrs. Smith reach New York. If she runs at a constant 25 miles per hour, how many miles does she run in total?
6. Galileo only watches 4 channels during a certain period. He spends one-third of the period watching the History Channel's documentary about him. He spends one-fourth of the time watching Jeopardy!, where he is the answer to the final jeopardy question. Then he watches ESPN for MathCounts for another one-fourth of the time. He spends the rest of the time, 7 hours, watching the A*nswer to Everything* channel. How long, in hours, is the period in which Galileo does all of these things?
7. If p represents a positive prime then what is the smallest p such that $p^4 + 5p^3$ is a perfect cube?
8. Mr. Holbrook cannot decide who to put on the 15-member Mu Teams. He has a list of 19 people that he will consider for it. Knowing that he has to put Chris, James, and Brian on it, what is the probability Mr. Holbrook will leave Josh off the team?
9. If Jim drives at 20 miles per hour for 40 miles and 40 miles per hours for 20 miles, what was the average speed, in miles per hour, Jim drove at for the entire trip?
10. Tom has forgotten to study for Mr. Galitsky's test. However, Tom knows physics very well, and gets an easy question correct all the time, and hard questions right 75% of the time. He finds that there are 10 easy questions and 10 hard questions on the test. The probability that he gets 19 questions correct is in the form $a \times \frac{b^c}{d^e}$ where a, b, and d are prime numbers. Find a + b + c + d + e.
11. What is the units digit of $(7^{15})^3$?
12. If f(x) = 6x-7 and g(x) = x-5, find g(f(x))-f(g(x)).
13. If the number of rabbits multiply by 5 in population every hour, and the number of ants multiply by 3 every hour, and rabbits initially have a population of 81 and ants have an initial population of 625, after how many hours will the rabbit population equal the ant population?

14. How many zeros does $\frac{30!}{4\times 5!}$ end in?

15. Ben, Brian, Chang, Eugene, and Michael are either ducks or geese. Ducks' statements are always false, while geese's statements are always true:

    Ben says Brian is a goose

    Chang says Eugene is a duck

    Michael says that Ben is not a duck

    Brian says Chang is not a goose

    Eugene says Michael and Ben are different animals.

    How many ducks are there in this group?

16. In order to decide where he is moving, Dr. Early plays a coin game against himself. If he gets the sequence THH first, he moves to Austin. If he gets the sequence HHH first, he moves to Tacoma. What is his chance of moving to Tacoma? (T represents tails and H represents heads)

17. Pyramid ABCDE has square base ABCD and edges AE, BE, CE, DE of equal length. If AB is 10 centimeters and the height of the pyramid is 10 centimeters, then compute the fourth root of the product of all edges of the pyramid. Express your answer in the form $a\sqrt{b}$ where b is not divisible by a perfect square.

18. We have 4 girls and 3 boys (all distinguishable) that wish to be seated in a 7-seat row at the movie theater. What is the probability that the two people at each end of the row were both boys or both girls? (Express your answer as a common fraction in lowest terms)

19. I have a paper plate with radius 9 cm which has a hole of radius 5 cm cut out of the center of it. I accidentally drop a coin with radius 2 cm that will randomly land somewhere on the plate (all of the coin is on the plate). What is the probability that it will drop in the hole completely (all of the coin is within the hole)?

20. Six problems (numbers 1 through 6) are given to students for a math test. A student can score 0, 1, 2 or 3 points for each problem. Compute the number of ways 15 points can be scored.

21. The polynomial $3x^2$-2x-2 has roots p,q. what is the value of $\frac{1}{p}+\frac{1}{q}$?

22. Professor Stevens wants to arrange the desks in his room. If he makes three rows with an equal numbers of desks, he has 1 desk left over. If he arranges the desks in 5 rows of equal numbers of desks, he has 3 desks left over. If he arranges the desks in 7 rows, he also has 1 desk left over. How many desks are there in the Professors room, given that he has less than 105 desks?

23. Geo draws two concentric circles with distinct radii. Then, he draws a chord on the larger circle so that the chord is tangent to the smaller circle. If the chord has length 14, then what is the area, in terms of π inside the larger circle but outside the smaller circle?

24. The River Styx flows along the line y=x. Starting from (0,0), Demeter can only move one unit in the

positive x direction or the positive y direction. She does not want to cross the Styx (hitting the line y=x does not count as crossing) on her way to the point (3,3). How many paths can Demeter take?

25. What is the coefficient of x^{99} in the expansion of $(x-1)(x-2)...(x-100)$?
26. The area of a circle circumscribed about a regular hexagon is 2π. What is the area of the hexagon?
27. Mr. Holbrook has two watches with 12 hour cycles (8 o'clock could mean both 8 a.m. or 8 p.m.). One watch gains a minute per day and the other loses one and a half minutes per day. If Mr. Holbrook set them both on the correct time, how many days will it take before they both tell the correct time again?
28. From a horizontal distance of 50 meters, the angles of elevation to the top and bottom of the Colossus at Rhodes are 45 and 30 degrees respectively. If the height of the Colossus is in the form $a\times(1-\frac{\sqrt{b}}{c})$, where b is not a multiple of a square, compute a + b + c. (Assume the object is perpendicular to the ground)
29. Jason brings his basketball on a fishing trip. He accidentally drops the ball of radius 13 centimeters into the water. Luckily, the density of the ball is less than that of the water, so the top of the ball remains 8 centimeters above the surface of the water. What is the circumference, in terms of π centimeters, of the circle formed by the contact of the water surface with the ball?
30. Chang, Brian and Eugene run a 2000 meter race. Chang finishes 200 meters ahead of Brian, and Chang is 290 meters ahead of Eugene. If Brian and Eugene both continue at their previous average speed, how many meters in front of Eugene will Brian finish?
31. Bob needs a date for Senior Prom. He is too shy to ask anyone in person, so he decides to send 5 Instant Messages at the exact same time to ask five different girls. He has a $\frac{9}{10}$ chance of being rejected by each girl. If the probability that Bob will be in trouble, (either having no date or having more than 1 date), is in the form $\frac{m}{n}$, what is n - m?
32. What is the area of the region in the Cartesian Plane that contains points (x,y) such that $|x|+|y|+|x+y|\le 2$?
33. In order to celebrate graduation year, Christine has decided to create a convex 2006-gon (a polygon with 2006 sides). Then, she picks a vertex, and draws lines connecting it to all vertices. How many non-degenerate regions (regions with area greater than 0) will she create by doing this?
34. FGHIJKLM is a cube with side length 1. P, Q, R, S, T, U are the midpoints of FI, FJ, JK, KL, LH, HI respectively. Compute the area of the hexagon PQRSTU .
35. Given that $K=1+\frac{1}{2+\frac{1}{K}}$ (K>0), and that K is expressible in the form $\frac{a+\sqrt{b}}{c}$ where a ,b, c are integers (b is not the multiple of a perfect square), compute a + 4b + 2c.
36. Harry assigns an integer to each of the six faces of the cube. Then, he assigns a number to each of the eight

vertices of the cube by adding the numbers of the faces that intersect at the vertex. Then, he adds up the eight vertex numbers. What is the largest number that is guaranteed to divide, that is to go into, this sum?

37. A rhombus has half the area of a square with the same side-length. What is the ratio of the long diagonal to the short diagonal in the rhombus? (Express the answer in the form $a+\sqrt{b}$)

38. Thermistocles is attempting to avoid ostracism. He is given 18 white balls and 6 black balls. He must divide them into three boxes, with at least one ball in each box. Then, he randomly selects one of the boxes and picks a ball from it. He only avoids ostracism if it is white. There is a most favorable and a least favorable distribution for Thermistocles. Compute the difference in probability of avoiding ostracism between the two distributions.

39. Archimedes loves circles. He randomly picks 2 points and then draws the chord between them. What is the probability the chord has length greater than the radius of the circle?

40. Joe decides to collect all 4 toys of distinct Star Wars characters. Every time he buys fries, he has an equal chance (25%) of collecting each one of four characters. What is the expected number of times (the expected value is just the inverse of the probability, so we expect to flip a coin twice to get a heads) he has to buy fries, expressed as a fraction in lowest terms, in order to collect all four characters?

41. Four spheres of radius 10 centimeters lie on a table, with their centers forming a square with four sides equal to 20 centimeters. If a fifth sphere, also of radius 10 centimeters, is to be placed on top of and touching the four spheres, what is the volume of the pyramid formed by the centers of the 5 spheres? (Express the answer as a fraction in simplest form with a radical in the numerator)

42. If $x_n^2 - x_{n-1}x_{n+1} = (-2)^n$ for $n \geq 1$, and $x_0 = x_1 = 1$, then compute x_3 .

43. A triangle ABC has AB=5, BC=6, and CA=7. Let I be the point where the angle bisectors meet and M be the point where the angle bisector of A meets side BC. If $\frac{MI}{IA}$ is in the form $\frac{m}{n}$, where m and n are integers, compute $m + n$.

44. New York and New Jersey are competing in a best-of-seven basketball series. The first team to win 4 games wins the series, and there are no ties. Given that the probability that NJ wins a game is 75%, and the probability that NJ wins in 6 games is expressible as $\frac{a}{b}$ in lowest terms, determine the number of factors of $a \times b$.

45. Let A(x), S(x), T(x) be distinct polynomials of the form ax^2+bx+c. The ordered triple (a, b, c) is selected without replacement from {1,2,3}. Let the sum of the roots of each polynomial be denoted by *p* and the product of the roots by *q*. We then find the ordered pair (p,q) for each of the polynomials. The three co-ordinates form a triangle. Find the maximum possible sum of the co-ordinates of the centroid of the triangle (the centroid is where the three medians intersect and is the center of mass of the triangle).

46. Josh wants you to calculate the radii of his red snooker balls. He knows that fifteen of them can be fitted tightly (outer balls are tangent to the frame) into an equilateral triangular frame of side length 876

millimeters (The triangular frame is higher than the radii of the balls). Express the radius in the form $\frac{a}{b+\sqrt{c}}$

47. Dr. Nevard has 16 distinguishable students in his class. If one of them falls asleep with probability 0.75, and the rest with probability .25, then if the probability that exactly 15 student stay awake is in the form $\frac{a^b}{c^d}$, where a and c are primes, what is a + b + c + d?

48. Engineers want to connect two parts of an unfinished road. The two parts are parallel, but end at points A and B respectively. We draw line segments AC and BC are perpendicular at point C with AC perpendicular to the two unfinished roads (BC is parallel). The engineers join the two parts of the road with two congruent (the same) circular curves. Each curve is tangent to *both* one part of the road and the other curve (at the midpoint of line segment AB). Given that AC is 1200 meters and BC is 900 meters, what is the radius of the two curves in meters?

49. Mr. Holbrook has devised a test to find any math geniuses at the Academy. 5% of students are actually math geniuses and they are correctly identified 95% of the time. The rest are not math geniuses, and they are correctly identified 95% of the time. If Jae takes the test, and the test says that he is a math genius, what is the probability that he actually is one?

50. The sum and the product of five positive integers x,y,z,u,v are equal. If $x \le y \le z \le u \le v$, how many distinct solutions (x,y,z,u,v) are there?

Diagram for #49