7^{th} Grade Competition

Bergen County Academies Math Competition 2006

22 October 2006

1. Mrs. Stone makes 20 telephone calls each day. How many telephone calls does she make in the month of January?
2. When the band The Beatles first formed, they had 1 fan. This fan told 3 other people about The Beatles. Each of these 3 people told 3 other people about The Beatles. As a result, how many people in total knew about The Beatles?
3. Every time Joey visits Monica and Chandler's apartment, he steals one more food item than he did the last time. If he visits once every day for a week, and steals 3 items the first day, how many food items does he steal in the week?
4. How many perfect squares are there between 2 and 140?
5. The sum of the ages of Sam and Cara is 30. Sam is 4 years younger than Cara. How old is Cara?
6. Find the product of the even multiples of 5 that are greater than 1 and less than 49.
7. Donald has twice as many books as Kevin. Kevin has 4 times as many books as Watson. Watson has 6 books. How many books do Donald, Kevin, and Watson have in total?
8. Compute $(1 + 2 + 3 + \cdots + 98 + 99) \div (.01 + .02 + .03 + \cdots + .98 + .99)$.
9. 9 is 15% of what number?
10. Find the probability that a randomly chosen positive whole number less than or equal to 90 is a multiple of 17.
11. Sixteen teams compete in a soccer tournament. Each game, one team wins and one team loses, and the losing team is eliminated. How many games must be played so that only one team remains undefeated?
12. Vincent likes to mix different types of soda. He buys one liter of cola for \$2.00, one liter of lemon-lime soda for \$1.50, and 2 liters of orange soda for a total of \$3.50. If he mixes these all together, what is the price per liter of the mixture?
13. Jen took pictures of exactly 40 birds. Exactly 30 of the birds were blue, and exactly 15 of the birds were male. How many of the birds were both blue and male?
14. Connie took a test with 25 questions. For every question she got right, she earned 4 points, and for every question she got wrong, she lost 1 point. She answered every question, and got a score of 80. How many questions did she get right?

15. How many 2-digit whole numbers contain the digit 2 exactly once?

16. Christine can sing 120 notes every minute, while Rachel can sing 180 notes every minute. If Christine and Rachel begin singing at the same time, how long, *in seconds*, will it take for their combined note total to equal 1080 notes?

17. Farmer John wants to buy fencing for the perimeter of his rectangular field. The length of the field is 17 meters and the area of the field is 119 square meters. Given that fencing costs 10 cents per meter, how much money will Farmer John have to spend?

18. Sally has 7 coins, which together make 72 cents. Each coin is a penny, nickel, dime or quarter. How many dimes does Sally have?

19. If a box of donuts is marked down 20% from its original price of $5, then marked back up 20% from the sale price, how much does this box of donuts cost now?

20. What is the number halfway between $\frac{1}{13}$ and $\frac{1}{9}$?

21. What is the positive difference between the sum of the first fifteen positive even numbers and the sum of the first fifteen positive odd numbers?

22. On planet Penev, the symbol $\left\{ \text{-} \right\}$ is defined as follows: $\left\{ \frac{x}{y} \right\} = (-1)^{x+y}$. Compute $\left\{ \frac{2}{4} \right\}$.

23. The point (10, 0) is 10 units from the origin (the point $(0, 0)$). Give a point, as an ordered pair (x, y), that is also 10 units from the origin such that both coordinates x and y are positive whole numbers and $x < y$.

24. Let $N = 20 \cdot 30 \cdot 50 \cdot 70 \cdot 90 \cdot 110 \cdot 130$. What is the smallest prime number that is not a factor of N?

25. If the length, width, and height of a box (rectangular prism) are all doubled, by how many times is the volume of the box increased?

26. Eddy scored 4 free throws out of the 13 free throws he has taken. How many consecutive shots must he now score in order to achieve a shooting percentage (free throws scored out of total free throws taken) of 50%?

27. How many lines of symmetry does a regular dodecagon (a polygon with 12 sides) have?

28. To complete his meal, James must select 3 different side dishes and one drink. There are 6 available different side dishes and 4 available drinks. The order in which James selects the side dishes and drinks does not matter. How many different meals can James select?

29. In the game of Gaussball, there are two ways to score points: a regular goal, worth 3 points; and a Gauss goal, worth 4 points. What is the greatest whole number that cannot be earned as a score in a game of Gaussball?

30. Three ducks - Duck 1, Duck 2, and Duck 3 - are sitting in a row. Duck 1 quacks once every six minutes. Duck 2 quacks once every nine minutes. Duck 3 quacks once every fifteen minutes. Given that all three ducks quack at 1:00 PM, when is the next time they will all simultaneously quack? ("Simultaneously" means at the same time.)

31. The angle measures of *acute* triangle ABC are all positive whole numbers. If $m\angle A = 50°$, what is the smallest possible measure, in degrees, of $\angle B$?

32. Compute $\left(\sqrt{2^3}\right)^{\frac{4}{3}}$.

33. Three children at a time can play jacks. For 30 minutes, 5 children take turns playing jacks so that each child plays for the same amount of time. How many total minutes does each child play jacks?

34. What is the units digit of 8^{22}?

35. A *palindrome* is a number that reads the same left-to-right as right-to-left. For example, 1234321 is a palindrome. How many palindromes between 1000 and 9999 are divisible by 5?

36. Dan runs at a speed of 1.1π miles per hour. How long, in hours, does it take Dan to run 2.75 times around a circle with radius 1 mile?

37. Dr. Mayers raises chickens and rabbits in his classroom. He counts 450 total animals, and 1050 total legs on the animals. Each chicken has 2 legs, while each rabbit has 4 legs. How many rabbits does Dr. Mayers have?

38. On a test, Jamie was asked to add 9 to a number x, and then divide the result by 2. However, Jamie accidentally *subtracted* 9 from x and then *multiplied* the result by 2. Luckily for her, she still got the correct answer! Find x.

39. Richard plays darts on a circular dartboard that is made up of two concentric circles (two circles with the same center but different sizes). Getting a bulls-eye means hitting anywhere in the inner circle, which has radius 2 inches. The outer circle has radius 3 inches. Richard always hits the dartboard, but does not always get a bulls-eye. If Richard has an equal chance of hitting each point on the dartboard, what is the probability that Richard gets a bulls-eye in his next shot?

40. If $2^x \cdot 3^x + 6^x = 432$, find x.

41. Aaron and John have a 300-square-foot lawn. It takes Aaron 30 minutes to mow the lawn alone, and it takes John 20 minutes to mow the lawn alone. If Aaron and John work together to mow the lawn at the same time, how long, in minutes, will it take them to mow it?

42. Four different numbers A, B, C and D are chosen from the set

$$\{-4, -\frac{3}{2}, -1, 0, \frac{5}{8}, 1, 4, 7\}.$$

What is the smallest possible value of the sum $\frac{A}{B} + \frac{C}{D}$?

43. Dr. Crane gave her class five homework assignments. Arthur only did three of these assignments. Dr. Crane picks two assignments at random to check, out of the five she gave. What is the probability that Arthur has done both of the assignments that Dr. Crane checks?

44. A cylindrical container of internal radius 2 inches and internal volume 32π cubic inches is $\frac{3}{4}$ filled with water. What is the maximum number of cylindrical disks of radius 2 inches and height $\frac{3}{5}$ inch that can be added to the container without letting the water overflow?

45. Jimbo, Jun, Jay, Jacob, Janine and Jenna are standing in a line with their backs to a wall. How many different orders could they be standing in so that Jenna and Jimbo are standing next to each other?

46. How many positive divisors does $(((2^0 \cdot 3^1)^2 \cdot 4)^3 \cdot 5)^4$ have?

47. A rectangle is divided into congruent squares. The sum of the perimeters of the squares is numerically equal to the area of the rectangle. What is the length of a side of one of the squares?

48. Ben and Joe are playing a game. They flip coins one at a time until either the sequence Heads-Tails or Tails-Tails comes up. If the sequence is Heads-Tails, Ben wins. If the sequence is Tails-Tails, Joe wins. What is the probability that Ben wins?

49. Each term in the Almost-Fibonacci sequence (except for the first two terms) is equal to the sum of the previous two terms. That is, $a_n = a_{n-1} + a_{n-2}$ for all $n \geq 3$, where a_n is the n^{th} term in the sequence. If the second term in the sequence is 4 and the ninth term is 110, what is the first term in the sequence?

50. What is the largest value of n such that $n^2 - 6n - 55$ is prime?