

5th Grade Competition and Solutions

Bergen County Academies Math Competition 2006

22 October 2006

1. To get ready for a math competition, Kevin buys a pencil for fifty cents, a ruler for one dollar, and graph paper for thirty dollars. How much money does Kevin spend?

Answer: \$31.50

Add the three costs, lining up decimal points:

$$\begin{array}{r} \$0.50 \\ \$1.00 \\ +\$30.00 \\ \hline \$31.50 \end{array}$$

2. If CNN is on channel 25 and YES is on channel 70, how many times does Michael have to hit the “channel up” button in order to switch from watching CNN to YES?

Answer: 45

Find the difference between 70 and 25: $70 - 25 = 45$.

3. Brian likes to feed the ducks. The first day he uses 1 slice of bread. The second day he uses 2 slices of bread. The third day he uses 3 slices of bread. If this pattern continues, how many slices of bread will he use on the tenth day?

Answer: 10

Follow the pattern: the fourth day he will use 4 slices of bread, and so on, until the tenth day when he will use 10 slices of bread.

4. Chang is illegally cruising down Route 4 in his father’s car. There are numbered markers along the route, each spaced one mile apart, which indicate how far one has travelled. If Chang starts at the 94th mile marker and the police stop him at the 117th mile marker, how many miles did Chang travel?

Answer: 23

Find the difference between numbers on the mile makers: $117 - 94 = 23$ miles.

5. $2 \times 0 \times 0 \times 6 =$

Answer: 0

Anything multiplied by 0 equals 0; thus, since there is at least one zero, the answer is 0.

6. Mrs. Stone makes 20 telephone calls per day. How many telephone calls does she make in the month of January?

Answer: 620

There are 31 days in January, and she makes 20 calls on each of these days, so she makes a total of $31 \cdot 20 = 620$ phone calls in January.

7. Calculate $10 - 9 + 8 - 7 + 6 - 5 + 4 - 3 + 2 - 1$.

Answer: 5

There are two approaches to this problem. We could simply carry out each addition and

subtraction to find the answer. Or we could group the numbers in a clever way that simplifies our computations, namely $(10 - 9) + (8 - 7) + (6 - 5) + (4 - 3) + (2 - 1)$. Following order of operations, we evaluate the expressions in parentheses first and find that the value of each is 1. So this equals $1 + 1 + 1 + 1 + 1$, which is 5.

8. Arthur accidentally added 7 to a number instead of multiplying that number by 7 on a math problem. His answer was 11. What should he have gotten as an answer?

Answer: 28

We can work backwards to find the answer. Since he added, we can reverse that by subtracting: the original number was $11 - 7 = 4$. He was supposed to multiply this by 7, and $7 \cdot 4 = 28$.

9. To be prepared, Yoonjoo wants to carry 10 pencils with her at all times during school. Every hour, she gives one pencil to a friend and loses one pencil. If school is 8 hours long, at least how many pencils must Yoonjoo bring to school to remain prepared?

Answer: 26

The first hour, she gives away a pencil and loses a pencil, which means that she is short 2 pencils. Since this occurs every hour, this means that at the end of the day she will be short 16 pencils. Thus she must bring at least $10 + 16 = 26$ pencils to school.

10. James's pet alligator has a body and a tail that together are 5 times as long as its head. From the tip of its nose to the tip of its tail the giant alligator is 300 meters long. How many meters long is its head?

Answer: 50

Since the body and tail are 5 times as long as the head, there are really 6 equal sections of the body that are the all the length of the head. So the head is $300 \div 6 = 50$ meters long.

11. Asena and Christine are playing a game. Asena asks Christine to pick a number. She then asks her to add 12 to that number, multiply her answer by 3, subtract 36 from her new answer, divide that answer by 6, and multiply this answer by 2. If Christine picks the number 10 in the beginning, what number does she have at the end?

Answer: 10

$10 + 12 = 22 \cdot 3 = 66 - 36 = 30 \div 6 = 5 \cdot 2 = 10$.

12. Aaron knows he has somewhere between 100 and 110 baseball cards in his collection. If he counts his cards 2 at a time, he has 1 card left over. If he counts his cards 5 at a time, he has 2 left over. How many cards are in Aaron's collection?

Answer: 107

Since Aaron has one card left over when he counts his cards 2 at a time, we know that he has an odd number of cards. We also know that he has 2 cards left over when he counts his cards 5 at a time. There are only two numbers between 100 and 110 that satisfy this last requirement: 102 and 107. However, we know that the number cannot be even, so Aaron must have 107 baseball cards in his collection.

13. Express $\frac{11}{5}$ in decimal form.

Answer: 2.2

Either divide directly: $11 \div 5 = 2.2$, or note that $\frac{11}{5} = \frac{10+1}{5} = 2 + \frac{1}{5} = 2 + \frac{2}{10} = 2.2$.

14. In Eddy's sock drawer, there are two red socks, three blue socks, and two green socks. If Eddy takes socks out of the drawer one at a time and does not put them back, how many must he take out so that he can make at least one matching pair?

Answer: 4

Suppose Eddy takes out three socks, and does not yet have a matching pair. This can only

happen if he takes out one red, one blue, and one green sock. The next (fourth) sock he takes out must then match one of the socks he has already taken out. If the first three socks were not all different colors, they must have contained a matching pair.

15. A military clock, instead of having the numbers 1 through 12, has the numbers 1 through 24 evenly spaced around the circular face. What is the number directly across from 19?

Answer: 7

We could simply draw the clock and match the numbers up. We could also note that numbers opposite each other will always have a difference of 12. So the number across from 19 is $19 - 12 = 7$.

16. For a class trip, Mr. Holbrook hires 2 buses, which can each hold 50 students. However, 123 students signed up for the trip. If a car can hold up to 4 students, what is the minimum number of cars needed in addition to the two buses to ensure that every student has a ride?

Answer: 6

The two buses hold $50 * 2 = 100$ students, leaving $123 - 100 = 23$ students for the cars. Each car holds a maximum 4 people so the 23 kids need at least $23 \div 4 = 5.75$ cars. Since cars are whole, this means that 6 cars are needed.

17. Two numbers have a sum of 15 and a product of 36. What is the larger of the two numbers?

Answer: 12

The two numbers must be 3 and 12, since $3 + 12 = 15$ and $3 \cdot 12 = 36$. The larger of these is 12.

18. In a high school physical education class, 11 students know how to climb a tree. 6 of these students are male. There are 14 boys total in the class. 7 girls in the class do not know how to climb a tree. How many students are there in this class?

Answer: 26

If 11 of the students in the class know how to climb a tree, and 6 are boys, that means that the other 5 are girls. 7 girls in the class do not know how to climb a tree, so there are $5 + 7 = 12$ girls in the class. Therefore the total number of students in the class is $12 \text{ girls} + 14 \text{ boys} = 26$.

19. $20 + 40 + 60 + 80 = (1 + 2 + 3 + 4) \times \underline{\quad ? \quad}$

Answer: 20

Each term on the left hand side of the equation is divisible by 20. Factoring out a 20 yields the expression $(1 + 2 + 3 + 4) \times 20$.

20. Mike arrived at a party 27 minutes before 3 : 23 PM. What time was it 11 minutes after he arrived?

Answer: 3:07 PM

27 minutes before 3 : 23 is 2 : 56. 11 minutes after 2 : 56 is 3 : 07.

21. If Eugene counted to 600 by 6's, starting with 6, how many numbers did he count that are less than 600?

Answer: 99

Eugene counted $600 \div 6 = 100$ numbers, all of which were less than 600 except for one, namely 600. So he counted $100 - 1 = 99$ numbers that are less than 600.

22. A triangle has a base of length 5 inches and a height to that base of length 8 inches. What is the area, in square inches, of the triangle?

Answer: 20

The area of a triangle is one half the length of its base times the length of its height to that base. So the area of this triangle is $\frac{5 \cdot 8}{2} = 20$ square inches.

23. If a bird's wings flap 64 times every second, then how many times do a bird's wings flap in 10 minutes?
Answer: 38,400
 There are 60 seconds in one minute, so there are $10 \cdot 60 = 600$ seconds in 10 minutes. Thus the bird flaps its wings $600 \cdot 64 = 38400$ times in 10 minutes.
24. If a race began at 3 : 43 PM and ended at 5 : 57 PM the same day, the race was half over at what time?
Answer: 4:50 PM
 The entire race was 2 hours and 14 minutes long. So the race was half over at the start time - 3 : 43 PM - plus 1 hour and 7 minutes, or 4 : 50 PM.
25. Will is 11 years old, and his sister is 7 years old. What will be the sum of Will's age and his sister's age when Will is 26 years old?
Answer: 48
 In $26 - 11 = 15$ years, Will will be 26, and his sister will be $7 + 15 = 22$. Thus the sum of their ages will be $26 + 22 = 48$.
26. Hannah goes to Boston Market to eat lunch. She picks a piece of chicken for her main meal, and must select a side dish and a drink. The possible side dishes are mashed potatoes, corn, green beans, and cinnamon apples. The possible drinks are Coca Cola, Sprite, orange soda, iced tea, lemonade, and water. How many different meals can Hannah select?
Answer: 24
 For each of the four side dishes, Hannah can pick any of the six drinks. So there are $4 \cdot 6 = 24$ total combinations.
27. If the same number of 3's, 5's, and 7's are added together to get a sum of 105, what is the total number of 3's used in the sum?
Answer: 7
 There must be $105 \div (3 + 5 + 7) = 105 \div 15 = 7$ of each.
28. Dan likes bowling, pool, and math. He decides to number the 10 bowling pins 1 through 10 and adds up the numbers on each pin, obtaining a number P . Then he numbers each of the 15 pool balls 1 through 15 and adds these up to obtain a number B . Find $B - P$.
Answer: 65
 We could find B and P separately by adding the numbers 1 through 10 and 1 through 15, and then we could find $B - P$. It is quicker to note that B is just $P + 11 + 12 + 13 + 14 + 15$ (since $P = 1 + 2 + \dots + 10$). Therefore, $B - P = 11 + 12 + 13 + 14 + 15 = 65$.
29. Compute $(1 + 2 + 3 + \dots + 98 + 99) \div (.01 + .02 + .03 + \dots + .98 + .99)$.
Answer: 100
 Note that multiplying each term in the divisor by 100 yields $1 + 2 + 3 + \dots + 98 + 99$. So the dividend is 100 times the divisor; thus the quotient is 100.
30. Ashley's calculator only performs one process: when she enters a number, the calculator shifts the entire number one decimal place to the right, adds 11, and returns the resulting number. For example, if she enters 11, the calculator shifts 11 a decimal place to the right to give 1.1, then adds 11 which gives 12.1. If she gets back 32.5 as an answer, what was her original number?
Answer: 215
 We reverse the calculator's process: $32.5 - 11 = 21.5$. Shifted one decimal place to the left, this is 215.

31. Caroline chooses a whole number greater than 1. She divides $1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7$ by that whole number. Then she divides the quotient by the same whole number. She continues dividing each new quotient by the same whole number until the remainder is no longer 0. Which whole number should Caroline choose so that she performs the division the greatest number of times?

Answer: 2

If we break the composite numbers in $1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7$ into prime factors, we see that there are four 2's in the prime factorization of this number, but only at most two factors of any other number. Thus Caroline can divide by 2 the greatest number of times.

32. Counting from either end of the line, Michelle is 18th in line. How many people are in the line?

Answer: 35

There are 17 people in front of Michelle and 17 people behind her. Including her, $17+17+1 = 35$.

33. Ian and Rob are having a barbecue. Ian goes to Shoprite and buys 62 ounces of meat. Rob goes to Pathmark and buys 82 ounces of meat. In total, how many *pounds* of meat did Ian and Rob buy? (There are 16 ounces in one pound.)

Answer: 9

Together they bought $62+82 = 144$ ounces of meat. Since one pound is equivalent to 16 ounces, they bought $144 \div 16 = 9$ pounds of meat.

34. Sixteen teams compete in a soccer tournament. Each game, one team wins and one team loses, and the losing team is eliminated. How many games must be played so that only one team remains undefeated?

Answer: 15

Fifteen teams must be eliminated, and it takes one game to eliminate one team, so 15 games must be played.

35. Dr. Mayers raises chickens and rabbits in his classroom. He counts 45 total animals, and 144 legs (each chicken has 2 legs, while each rabbit has 4 legs). How many rabbits does Dr. Mayers have?

Answer: 27

Suppose all the animals are rabbits. Then there would be $4 \cdot 45 = 180$ legs. If we substitute a chicken for a rabbit, we reduce the number of legs by 2. We want to reduce the number of legs by $180 - 144 = 36$, so we must substitute in $36 \div 2 = 18$ chickens. This means there are $45 - 18 = 27$ rabbits.

36. What is the remainder when 6668 is divided by 6?

Answer: 2

We know that 6666 is exactly divisible by 6, so 6668 is 2 more than a multiple of 6, and will have a remainder of 2 when divided by 6.

37. Sally has 7 coins, which together make 72 cents. Each coin is a penny, nickel, dime or quarter. How many dimes does Sally have?

Answer: 1

Sally must have 2 pennies, since every other coin has a cent value that is a multiple of 5. If her other 5 coins are all dimes, she would only have 52 cents, so she must have at least one quarter. If her other 4 coins are all dimes, she would only have $25 + 40 + 2 = 67$ cents, so she must have at least two quarters. If her other 3 coins are all dimes, she would have $50 + 30 + 2 = 82$ cents, which is too much, so at least one coin must be a nickel. This would yield $50 + 20 + 5 + 2 = 77$ cents, which is still too much, so two coins must be nickels. Thus she has 2 pennies, 2 quarters, 2 nickels, and 1 dime ($2 \cdot 1 + 2 \cdot 25 + 2 \cdot 5 + 1 \cdot 10 = 72$).

38. 9 is 15% of what number?

Answer: 60

Divide 9 by 15%: $9 \div 0.15 = 60$.

39. Connie took a test with 25 questions. For every question she got right, she earned 4 points, and for every question she got wrong, she lost 1 point. She answered every question, and got a score of 80. How many questions did she get right?

Answer: 21

Guess-and-check can be used to solve this problem. Alternately, let c be the number of questions Connie got right. Then she got $25 - c$ questions wrong. So her score is $4c - (25 - c) = 5c - 25 = 80$. Solve for c : $5c = 105 \Rightarrow c = 21$.

40. If there are 5 zounds to a zong, 7 zongs to a zig, and 3 zigs to a zap, how many zounds are there to 3 zaps?

Answer: 315

There are $5 \cdot 7 = 35$ zounds to a zig, and since there are 3 zigs to a zap, there are $35 \cdot 3 = 105$ zounds to a zap. So there are $3 \cdot 105 = 315$ zounds to 3 zaps.

41. In 4 ponds, there are 33 ducks. For any arrangement of ducks in these ponds, you can find at least one pond with at least n ducks. What is the largest possible value of n ?

Answer: 9

This is an application of the Pigeonhole Principle, with 33 “pigeons” (ducks) in 4 “holes” (ponds). Since $33 \div 4 = 8.25$, we round up to get the answer of 9.

42. Ben, Joe, Rob and Mike like to play ping-pong after math team. Each played exactly 3 games of ping-pong with each of the others. How many games of ping-pong were played altogether?

Answer: 18

If we label the boys B, J, R, and M, we know that there are 6 games if they each played each other once: BJ, BR, BM, JR, JM, RM. But since they each played 3 games against each other, they played $6 \cdot 3 = 18$ games total.

43. Rachel begins writing the non-negative whole numbers (0, 1, 2, ...) across a sheet of paper in increasing order, with no spaces between them. She stops when she reaches the end of the paper. There is only room across the paper for 100 digits. What is the last digit Rachel writes?

Answer: 4

The numbers 0 – 9 take up 10 spaces. Since there are 90 spaces left and we know that the next set of numbers consist of two digits each, we will be able to fit 45 two-digit numbers, beginning with 10. The 45th two-digit number is $10 + 45 - 1 = 54$, so the last digit in the row is 4.

44. A can of mango juice concentrate is mixed with 3 cans of water to make $1\frac{1}{2}$ pints of mango juice. Each can of water is the same size as the can of mango juice. What fraction of a pint does one can hold?

Answer: $\frac{3}{8}$

If 4 cans can hold $1\frac{1}{2}$ pints, which is the same as $\frac{3}{2}$ pints, then 1 can can hold $\frac{1}{4} \cdot \frac{3}{2} = \frac{3}{8}$ of a pint.

45. The sum of five different positive integers is 100. What is the largest possible value for any one of these integers?

Answer: 90

In order to find the largest number, the other four numbers must be as small as possible. The four smallest distinct positive integers are 1, 2, 3 and 4. Thus the largest possible value for one of the integers is $100 - (1 + 2 + 3 + 4) = 100 - 10 = 90$.

46. How many perfect squares are there between 2 and 140?

Answer: 10

The perfect squares between 2 and 140 are $2^2 = 4, 3^2 = 9, \dots, 11^2 = 121$. So there are $11 - 2 + 1 = 10$.

47. Sujin paints $\frac{1}{4}$ of a pole green. Christine then paints $\frac{2}{3}$ of the pole red, and leaves the remaining 2 feet unpainted. How many feet long is the entire pole?

Answer: 24

$\frac{1}{4} + \frac{2}{3} = \frac{3}{12} + \frac{8}{12} = \frac{11}{12}$. The remaining $1 - \frac{11}{12} = \frac{1}{12}$ of the pole is 2 feet long; thus the entire pole is $2 \cdot 12 = 24$ feet long.

48. A *palindrome* is a number that reads the same left to right as it does right to left. For example, 123321 is a palindrome. How many palindromes are there greater than or equal to 1 and less than 100?

Answer: 18

Every one-digit number reads the same forwards and backwards. A two-digit number reads the same forwards and backwards if and only if both of its digits are the same. So the palindromes between 1 and 100 are the 18 numbers 1, 2, ..., 9, 11, 22, ..., 99.

49. The average (arithmetic mean) of five numbers in a list is 35. The average of the first two numbers is 26. What is the average of the last three numbers?

Answer: 41

The sum of all the numbers in the list is $5 \cdot 35 = 175$. The sum of the first two numbers is $2 \cdot 26 = 52$, so the sum of the last three numbers is $175 - 52 = 123$. The average of these numbers is $123 \div 3 = 41$.

50. How many positive divisors does 5040 have?

Answer: 60

First find the prime factorization of 5040: $2^4 \cdot 3^2 \cdot 5 \cdot 7$. The number of divisors is the product of one more than each of the exponents; that is, $(4 + 1) \cdot (2 + 1) \cdot (1 + 1) \cdot (1 + 1) = 5 \cdot 3 \cdot 2 \cdot 2 = 60$ divisors. The number 5040 is considered a *highly composite number*, which means it has more divisors than any number beneath it. In fact, this number has every highly composite number beneath it as a factor.