

6th Grade Competition

Bergen County Academies

21 October 2007

1. A student is compiling questions for a math competition. There are five levels of competition, each with 50 questions. No question can be repeated. How many questions does she need?

Answer: 250

She needs $50 * 5 = 250$ questions to complete the competition.

2. What is the probability of choosing a vowel from the alphabet?

Answer: 5/26

There are 5 vowels and 26 total letters in the alphabet. Hence the probability is $5/26$.

3. Mark writes three questions Monday, five questions on Tuesday, seven questions on Wednesday, and so on. How many questions will he have written, in total, at the end of Sunday?

Answer: 63

He would have written $3 + 5 + 7 + 9 + 11 + 13 + 15 = 7 * 9 = 63$ questions.

4. An adult working alone requires three hours to do a certain job. A child working alone requires six hours to do the same job. How many hours will it take the adult and child, working together, to do this job?

Answer: 2

The adult can finish $\frac{1}{3}$ of the job per hour. The child can finish $\frac{1}{6}$ of the job per hour. Together, they can finish $\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$ of the job per hour. Thus, it would take them 2 hours to finish.

5. Find the sum of the numbers from 1 to 15 inclusive.

Answer: 120

$1 + 2 + 3 + \dots + 14 + 15 = (1 + 15) + (2 + 14) + (3 + 13) + \dots + (7 + 9) + 8 = (7 * 16) + 8 = 120$

6. 10% of the marbles in Emily's bag are blue. Emily has 50 marbles. How many are blue?

Answer: 5

10% of 50 = $0.1 * 50 = 5$ marbles are blue.

7. Let $A = 1, B = 2, \dots, Z = 26$. What is $A + T + T + I + T + U + D + E$?

Answer: 100

The answer is $1 + 20 + 20 + 9 + 20 + 21 + 4 + 5 = 100$.

8. My father is 4 times as old as I. In 20 years, he will be twice my age. How old is he now?

Answer: 40

If x is my age, $4x$ is father's age so $4x + 20 = 2(x + 20)$ so $x = 10$ and father is 40.

9. The peel of a banana weighs about $1/8$ of the total weight of the banana. If you buy 3 kg of bananas at a price of \$1.20 per kilogram, how much are you paying for the inside of the bananas?

Answer: \$3.15

$\$1.20 * 3/8 = \0.45 is spent on the peel, and $\$3 * 1.20 - \$0.45 = \$3.15$ on banana.

10. A train is traveling at the rate of 1 mile per 1 minute and 20 seconds. If the train continues at this rate, how many miles will it travel in one hour?
Answer: 45
 At this rate, the train will go 3 miles in 4 minutes. Since there are 15 4-minute periods in an hour, the train will travel 15 3-mile lengths, or 45 miles.
11. Find the 21st term in the sequence 2, 4, 6, 8, ...
Answer: 42
 In general, the n th term is $2 * n$. Hence, the 21st term is $21 * 2 = 42$.
12. Snow White bought 1 t-shirt for each of the seven dwarves for \$9.95 each. The cashier charged her an additional \$13.07 in sales tax. She left the store with \$7.28. How much money did she start with?
Answer: \$90.00
 She started with $7 * \$9.95 + \$13.07 + \$7.28 = \90.00 .
13. A bag of marbles can be divided in equal shares among 2, 3, 4, 5, or 6 friends. What is the least number of marbles that the bag could contain?
Answer: 60
 The least common multiple of 2, 3, 4, 5, and 6 is $2^2 * 3 * 5 = 60$.
14. How many zoinks can you buy with 300 zonkers knowing the exchange rate is 20 zoinks per zonker?
Answer: 6000
 $300 \text{ zonkers} \times 20 \text{ zoinks} / 1 \text{ zonker} = 6000 \text{ zoinks}$
15. What is the 10th term in the sequence 1, 5, 9, ...?
Answer: 37
 The first term is $1 \times 4 - 3$, the second is $2 \times 4 - 3$, the third is $3 \times 4 - 3$, and so on. Thus, the tenth term is $10 \times 4 - 3 = 37$.
16. If you put aside \$2.00 on January 1, \$4.00 on February 1, \$8.00 on March 1, \$16.00 on April 1, and so on, how much money would you save in six months?
Answer: \$126.00
 $2 + 4 + 8 + 16 + 32 + 64 = 126$.
17. In parallelogram $ABCD$, $AB = x + 8$, $BC = 3x$, and $CD = 4x - 4$. What is the perimeter?
Answer: 48
 $AB = x + 8 = 4x - 4 = CD$ so $x = 4$ and $BC = 3 * 4 = 12 = AB$ so the perimeter is $2 * (AB + BC) = 48$.
18. How many degrees are in the measure of the smaller angle that is formed by the hands of a clock when it is 4 o'clock?
Answer: 120
 Each hour amounts to $360/12=30$ degrees, then 4 hours will give rise to an angle of $30*4=120$.
19. An omelet requires 2 eggs and 1 pepper. I have 12 eggs and 8 peppers. How many whole omelets can I make?
Answer: 6
 I have enough eggs for 6 omelets and peppers for 8 omelets, so I can make $\min\{6, 8\} = 6$.
20. A parking lot has space for 1000 cars. $2/5$ of the spaces are handicapped spaces. There were 200 handicapped cars and some non-handicapped cars in the parking lot. The parking lot was

$3/4$ full. How many non-handicapped cars were in the parking lot?

Answer: 550

$750 - 200 = 550$ non-handicapped cars.

21. Two weeks ago, stores A and B sold shirts at \$10. Last week, store A raised prices by 15% while store B reduced prices by 15%. Realizing their mistakes, yesterday store B raised prices by 15% while store A reduced prices by 15%. What is the difference in prices today?

Answer: 0

Store A's prices rose to $10 * 1.15 = 11.5$ and dropped to $11.5 * 0.85 = 9.775$ while store B's prices dropped to $10 * 0.85 = 8.5$ and rose to $8.5 * 1.15 = 9.775$.

22. Riding their bicycles, Alex and Brian leave from two different places at the same time and ride directly toward each other. Alex rides at 10 mi/h and Brian rides at 8 mi/h. If they meet after 40 minutes of riding, how far apart were they at the beginning?

Answer: 12

Each hour, Alex and Brian get $10 + 8 = 18$ miles closer to each other. If it only takes $2/3$ of an hour for them to meet, then their original distance was $(2/3)(18 \text{ miles}) = 12$ miles.

23. The average of seven numbers is 49. If 1 is added to the first number, 2 is added to the second number, 3 is added to the third number and so on, what is the new average?

Answer: 53

$49 + (1 + 2 + 3 + 4 + 5 + 6 + 7)/7 = 49 + 4 = 53$

24. The length of a rectangle is four times as long as its width. The area of the rectangle is 100 metres squared. What is the length of the rectangle?

Answer: 20

If x is the width, $4x$ is the length, so $x * 4x = 100 \Rightarrow x * x = 25 \Rightarrow x = 5$.

25. Homer Simpson entered a pie eating contest. Determined to win, he trained for 6 days. Each day he ate 4 more pies than the day before. Homer ate 150 pies while in training. How many pies did he eat on the 6th day?

Answer: 35

If he ate x pies on the 6th day, he ate $x - 4$ days on the 5th day, $x - 2 * 4$ days on the 4th day, ... and $x - 5 * 4$ days on the first day. He ate $6 * x - 60 = 150$ pies during training. $x = 35$

26. A *palindrome* number, such as 22 or 565, is the same when written forwards or backwards. How many palindromes are there that are less than 1000?

Answer: 99

There are 9 2-digit palindromes and 90 3-digit palindromes, which gives 99.

27. The number of hours that were left in the day was one-third of the number of hours already passed. What time is it?

Answer: 6:00 PM

Let x be the number of hours left in the day. Then, $3x$ is the number of hours passed, so there are $4x$ hours in the day. $4x = 24$ so $x = 6$ and the time is 6:00 PM.

28. My change purse contains 16 coins in dimes and quarters. If the value is \$2.50, how many dimes are there?

Answer: 10

Let x equal the number of dimes. Then, $16 - x =$ the number of quarters, $10x =$ value of the dimes in cents, and $25 * (16 - x) =$ value of quarters in cents. $10x + 25 * (16 - x) = 250$ so $15x = 150$ and $x = 10$.

29. Sophia receives a 15% discount on an item. She has a 10% coupon as well. What is the total percent discount?

Answer: 23.5%

Supposing the old price of the item is \$100, the first discount is $\$100 * .15 = 15$ and the new price is $\$100 - \$15 = \$85$. The second discount is $\$85 * .10 = \8.5 , so the final price is $\$85 - \$8.50 = \$76.50$. The net discount is $(100 - 76.50)/100 = 23.5\%$.

30. Bob, Don, Alice, Carol, and Eve want to sit at a circular table so that the boys do not sit next to or across from each other. If Alice must sit next to two girls, how many possible arrangements are there?

Answer: 0

Supposing we go counterclockwise starting from B, we must have B _ D _ _ or B _ _ D _ . There is no good place for Alice to sit since the remaining seats in all cases are next to a guy.

31. The vertices of a triangle are at (3,1), (8,1) and (8,3). What is the area of the triangle?

Answer: 5

The vertices form a right triangle with a base of 5 and a height of 2. Hence the area is $A = 1/2 * b * h = 1/2 * 5 * 2 = 5$.

32. Mr. Teacher asked the children to find the consecutive pages in their math books whose page numbers add up to 85. What is the larger of the two pages?

Answer: 43

The smaller number is one less than the larger number, so the larger number is $(85 + 1)/2 = 43$.

33. A decade ago, I bought a car for \$40,000. I have since paid \$5,000 a year for maintenance and \$0.10 per mile for gas. I have driven 10,000 miles per year. How much have I spent on my car since I purchased it?

Answer: \$60,000

I spent $\$5,000 * 10 + \$0.10 * 10000 * 10 = \$60,000$

34. A manufacturer claims that a new motor oil saves 5% of the gasoline used by a car. You and a friend have identical cars and drive the same 21,000 kilometres per year. You use the new motor oil but your friend does not. If your friend gets 42 kilometers per gallon, how many gallons of gasoline do you save in one year?

Answer: 25

If your friend get 42 kilometers per gallon, your friend would use $21,000/42 = 500$ gallons per year. If you save 5%, you save $500 * 0.05 = 25$ gallons.

35. The average of a set of 5 numbers his 32. The number 132 is removed from the set. By how much is the mean reduced?

Answer: 25

Let y be the sum of the 4 remaining elements, then $(y + 132)/5 = 32$, hence $y = 5 * 32 - 132 = 28$. Therefore the new mean is $y/4 = 7$. The answer is $32 - 7 = 25$.

36. Find the 6th term in the geometric sequence 3, 6, 12, ...

Answer: 96

The ratio is 2, so we have to multiply each term by 2 to find the next one. So, the fourth term is $12 * 2 = 24$, the fifth term is $24 * 2 = 48$, and the sixth term is $48 * 2 = 96$.

37. The measure of the supplement of an angle is 4 times the measure of its complement. Find the measure of the angle, in degrees.

Answer: 60°

Let x° be the angle's measure. Then $(180 - x)^\circ$ is the supplement's measure, $(90 - x)^\circ$ is the complement's measure. $180 - x = 4(90 - x) \Rightarrow x = 60^\circ$

38. How many terminating zeroes does the number $(215)(34)(59)(710)$ have?

Answer: 2

The prime factorization of this number is $(5 * 43)(2 * 17)(59)(2 * 5 * 71)$. For a number to have a terminating zero, it must have a multiple of ten as a factor. Since $10 = 2 * 5$, we look for pairs of 2's and 5's in the prime factorization of the number. There are two 2 - 5 pairs, so the number is divisible by $10^2 = 100$, so it has two terminating zeros.

39. Mr. Teacher awards extra credit points to his students with test grades out of 100 that are below the average of the 6th grade class. If 73 students take the same test, what is the largest number of students who can be awarded extra credit?

Answer: 72

The largest number of students who can be awarded extra credit happens when everyone else has equal scores except one who has a score that is more than everyone else. So the answer is $73-1=72$ (72 students scoring 50 and one scoring 100, for example).

40. Watson assigns consecutive letters of the alphabet increasing consecutive integer values, starting from A. If $H + K + L + P = 2007$, then what is Z ?

Answer: 516

$H = A + 7, K = A + 10, L = A + 11, P = A + 15$ so $H + K + L + P = 4A + 43 = 2007$ and $A = 491$, so $Z = A + 25 = 516$.

41. In Flatland, there are four types of coins. A zonk is worth 10 bonks. A conk is worth 8 bonks. A donk is worth 10 zonks. I have 227 bonks. What is the smallest number of coins I could have?

Answer: 6

The optimal configuration is 2 donks, 1 zonk, 2 conks and 1 bonk, for a total of 6 coins.

42. Three numbers are chosen from the set $\{1, 2, 3, 4, 5, 6\}$ and added. What are all of the possible sums?

Answer: {6, 7, 8, 9, 10, 11, 12, 13, 14, 15} OR 6 to 15 or [6-15]

Observe that the sum is always an integer, and the sum must be between $1 + 2 + 3 = 6$ and $4 + 5 + 6 = 15$. All intermediate values can be obtained.

43. How many factors does the number 540 have?

Answer: 24

$540 = 2^2 * 3^3 * 5$. So, in order to find the number of factors of 540, we need to know the number of choices for the exponents of 2, 3, and 5. There are three choices for the exponent of 2: 0, 1, or 2. Similarly, there are 4 choices for the exponent of 3: 0, 1, 2, and 3. There are two choices for the exponent of 5: 0 or 1. To find the total number of factors, we multiply $3 * 4 * 2 = 24$ factors.

44. Farmer John has 20 yards of fence. He wants to create a rectangular pen for his cows. What is the largest area, in square yards, that he can enclose?

Answer: 25

Suppose the pen's width is x . Then, the pen's length is $10 - x$ and the area is $x * (10 - x)$. The maximum value occurs when $x = 5$, so the area is 25 square yards.

45. Find the only four-digit number that starts with the digit 7 and is divisible by 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10.

Answer: 7560

The least common multiple of 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 is $2^3 * 3^2 * 5 * 7 = 2520$. But the required number must begin with a 7, so it is $2520 * 3 = 7560$.

46. A drawer contains 3 red socks, 5 blue socks, and 7 yellow socks. Without looking, what is the smallest number of socks that must be chosen so that you can be sure that you have at least three socks of each color?

Answer: 15

Notice that if only 7 socks are chosen, they could all be yellow. Similarly, if only 12 socks were chosen, they could all be blue or yellow. And if 14 socks were chosen, they could be 2 red socks, 5 blue socks, and 7 yellow socks, which would not be at least three of each sock. Thus, one would have to pick all $3+5+7=15$ socks to be sure that he has at least three socks of each color.

47. A convex polygon has n sides and $9n$ diagonals. Find the value of n .

Answer: 21

The number of diagonals in a convex n -sided polygon is $n * (n - 3) / 2$. $n * (n - 3) / 2 = 9 * n$, so $n - 3 = 2 * 9$ so $n = 2 * 9 + 3 = 21$.

48. If $x = 9$, compute the value of $x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1$.

Answer: 1,000,000

Notice that $x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1 = (x + 1)^6 = (9 + 1)^6 = 10^6 = 1,000,000$

49. What is the units digit of $1^{1000} + 2^{1000} + 3^{1000} + 4^{1000} + 5^{1000} + 6^{1000}$?

Answer: 5

Term by term, follow the patterns for the last digit of each term:

1^n : 1, 1, 1, 1, 1, 1, 1, 1, ... 2^n : 2, 4, 8, 6, 2, 4, 8, 6, ... 3^n : 3, 9, 7, 1, 3, 9, 7, 1, ...

4^n : 4, 6, 4, 6, 4, 6, 4, 6, ... 5^n : 5, 5, 5, 5, 5, 5, 5, 5, ... 6^n : 6, 6, 6, 6, 6, 6, 6, 6, ...

Each sequence repeats after 4 elements, so we just need to find the 1000th term of each sequence.

The sum of the units digits is $1 + 6 + 1 + 6 + 5 + 6 = 25$, so the units digit of the sum is 5.

50. During a grim medieval battle, 85% of the warriors lost an ear, 80% lost an eye, 75% lost an arm, and 70% lost a leg. What is the smallest percentage possible of combatants who lost all of the above?

Answer: 10 or 10%

Since 85% of the warriors lost an ear and 80% an eye (a total of 165%), at least 65% of them must have lost both an ear and an eye. Since at least 65% of the warriors lost both an ear and an eye, and 75% lost an arm (a total of 140%), at least 40% of them must have lost an ear, an eye, and arm and a leg each. Since at least 40% of the warriors lost an ear, an eye, and an arm and 70% lost a leg (a total of 110%), at least 10% of them must have lost an ear, an eye, and arm and a leg each.