7th Grade Competition

Bergen County Academies Math Competition
19 October 2008

1. Before taking the USAMO, a student notices that he has a bag of Doritos, two bags of Fritos, a can of soda, two bags of Skittles, two bags of Cheetos, and three Reese's cups. Each bag of Doritos has 130 calories, each bag of Fritos has 120 calories, each can of soda has 140 calories, each bag of Skittles has 60 calories, each Reese's cup has 30 calories, and each bag of Cheetos has 110 calories. If, during the test, the student consumes everything on his desk, how many calories will he have consumed?

2. $7+77+777+7777+77777 = ?$

3. Find the perimeter of an equilateral triangle with side length 9.

4. Josh and Kun-Soo have 2008 Frosted Flakes each. Josh gives Kun-Soo half of his flakes. After this, Kun-Soo gives all of his flakes to Josh. How many flakes does Josh have now?

5. If each minute of music takes up .9 megabytes of memory, then how many megabytes will a library containing 1440 minutes of music take up?

6. David Rush is in a hurry to get to class. His dorm at Philips Exeter is 300 meters away from his first class. He runs the first half of the distance at a brisk 5 meters per second. After this, he (instantly) slows down to 2 meters per second and finishes running at this rate. How many seconds did it take him to get to class?

7. A palindrome is a number that is read the same way forwards and backwards. Find the next largest palindrome after 80908.

8. Using the approximation 1 mile equals 1.6 kilometers, how many miles is 80.8 kilometers equivalent to?

9. Starting with the letter D, a large crowd of students repeatedly chants the letters D, B, and R in that order. Assuming the chant lasts long enough, which letter will be the 2008^{th} letter chanted by the students?

10. Evaluate $(1^2-0^2)+(2^2-1^2)+(3^2-2^2)+(4^2-3^2)+\ldots+(10^2-9^2)$.

11. Matt's suitcase weighs too much to put on the airplane – 56 pounds and 3 ounces. If the maximum allowable weight for a suitcase is 50 pounds, and one pound equals 16 ounces, then what is the minimum number of ounces of luggage Matt must remove from his suitcase before flying?

12. Solve for x: $|x-|3x||=6$.

13. Hannah is cramming for the SAT, and needs to memorize 300 words. She can cram 50 words each morning, but will forget 40 of them the next night. After how many mornings of studying will Hannah be able to write the correct definitions of all 300 words?

14. A cylinder with radius 8 and height 17 is rolled along the floor so that it completes 2 full revolutions. What is the area of the region of the floor that was touched by the cylinder during the roll?

15. Find $(2+3\imath)-(3-4\imath)$.

16. In how many ways can Ally, Brett, Candy, and Doug be lined up so that Ally is standing next to her best friend Doug?

17. If 9253824 beads are split among 12 people so that each person has the same whole number of beads, how many are left over?

18. In how many ways can 10 be written as a sum of three (not necessarily distinct) positive integers?

19. A group of people are trying to sit themselves in rows. If they sit in rows of four, there are three people left over. If they sit in rows of five, there are four people left over. If they sit in rows of six, there are five people over. What is the smallest possible number of people in this group?

20. While on page n of her book, Alice realizes that the product of the page number before this page (page $n-1$) and the product of the page number after this page (page $n+1$) is 168. What is n?

21. Austin runs an animal hospital that takes care of cats and dogs. The cats here only have three legs and one tail each and the dogs have six legs each and two tails. Victoria, the hospital inspector, walks in one day and counts 36 legs and 12 tails. If n is the number of animal heads in the hospital, find the sum of all possible values of n. There may be zero cats or zero dogs.

22. Two congruent circles with radii 6 have centers that are $6\sqrt{3}$ units apart. What is the area of the union of the two circles?

23. How many perfect squares have three digits or less?

24. I have only quarters, dimes, nickels, and pennies in my pocket. I count 93 cents in my pocket. What is the difference between the largest number of coins I could possibly have and the smallest number of coins I could possible have? There might be none of a certain type of coin.

25. Find the sum of all positive composite numbers less than or equal to 20.

26. Convert 3018, a number in base 10, to base 16. Give your answer without the subscript 16.

27. Let $x^{\underline{n}} = x(x-1)(x-2)\dots(x-n+1)$. Find $8^{\underline{3}}$.

28. Tom makes half of his 3-point shots. What is the probability that Tom makes at least five out of nine 3-point shots?

29. Compute the ratio of the area of a square to that of its inscribed circle.

30. If $3a-2b=11$ and $5a+7b=163$, find $(a+b)^2$.

31. If a, b, and c are positive real numbers such that $(a-b)^2+(b-c)^2+(c-a)^2=0$ and $a+b+c=9$, then find the product abc.

32. If $f(x) = \dfrac{1}{10x-5}$ for $1 \le x \le 10$, then find the maximum value of $f(x)$ on this range minus the minimum value of $f(x)$ on this range.

33. Evaluate $\sum_{n=1}^{10} n^2$.

34. A cubic ice cube weighs 920 grams. If the density of ice is .92 g/cm^3, then at least how many such ice cubes would it take to build a tower that is three meters tall? There are 100 cm in a meter.

35. Sam and Sherry are playing a game of Brawl using their favorite characters, Pit and Olimar respectively. Sam's percentage probability of winning is 55% while Sherry's percentage probability of winning is 45%. After two one-on-one matches, what is the probability that each has won a match?

36. Two positive integers are relatively prime if they share no factors other than 1. $\phi(n)$ is the defined as the number of numbers less than or equal to n that are relatively prime to n. For example, $\phi(6) = 2$. Find $\phi(35)$.

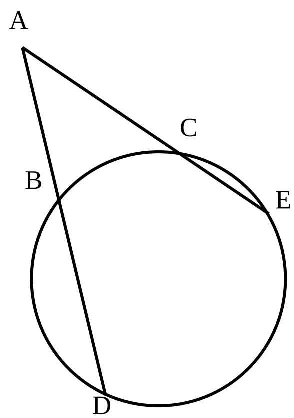
37. How many integer solutions (a, b), with $a < b$, does the equation $ab + a + b = 17$ have?

38. $n! = n(n-1)(n-2)\ldots 3\cdot 2\cdot 1$. Find the greatest integer a such that 50! is divisible by 2^a.

39. Find the ratio of volume to surface area of a cube with side length 18.

40. What is the length of the median to the hypotenuse in a $6-8-10$ right triangle?

41. In the drawing, $\overline{AB} = 7$, $\overline{AD} = 16$, and $\overline{AC} = 8$. Compute $\overline{CE}$.



42. In $\triangle ABC$, $\overline{AB} = 9$ and $\overline{BC} = 8$. If $\angle ABC = 30^o$, then what is the area of $\triangle ABC$?

43. Compute $\sqrt{2+\sqrt{2+\sqrt{2+\ldots}}}$.

44. $\lfloor x \rfloor$ is defined as the greatest integer less than or equal to x. For example, $\lfloor 5.5 \rfloor = 5$, $\lfloor \pi \rfloor = 3$, and $\lfloor 4 \rfloor = 4$. Compute the values of x for which $\lfloor x\lfloor x\rfloor + x\rfloor = 2$.

45. Each of three congruent circles is externally tangent to the other two. If the radius of each circle is 4, then find the area of the region in the middle of the three circles.

46. The real numbers a_1, a_2, a_3, ..., a_{20} are written in that order around a circle. Given that $a_1 = 1$, $a_{11} = 2$, $a_{14} = 3$, and that the sum of any four consecutive terms is 20, find a_4.

47. If $x^3 + ax^2 - 6x - 12$ is evenly divisible by $x - 1$, then find a.

48. Compute $\log_{10}\left(\frac{1}{2}\right) + \log_{10}\left(\frac{2}{3}\right) + \log_{10}\left(\frac{3}{4}\right) + \ldots + \log_{10}\left(\frac{999}{1000}\right)$.

49. If there are 6,000,000,001 people in the world and each has less than 100,000 hairs on his or her head, find the maximum number of people that MUST have the same number of hairs on their head.

50. Square $ABCD$ has side length 4. Points E and F are on sides $\overline{AB}$ and $\overline{BC}$, respectively, such that $\frac{\overline{AE}}{\overline{EB}} = 2$ and

$\dfrac{\overline{BF}}{\overline{FC}} = 3$. Find the difference between the area inside the square but outside both $\triangle CDE$ and $\triangle ADF$ and the area inside both $\triangle CDE$ and $\triangle ADF$.