7th Annual Bergen County Academies Math Competition

Fourth Grade

Sunday, 18 October 2009

1 Rules

1. You may use space on your test paper and additional scrap paper to do work. Your answers must be written on the answer sheet. We will not look at answers written on your test paper.
2. Each problem has only one answer. If you put more than one answer for a problem, you will be marked wrong. When changing an answer, be sure to erase or cross out completely.
3. Write legibly. If the graders cannot read your answer, it will be marked incorrect.
4. Fractions should be written in lowest terms. For example, if the answer is $\frac{1}{2}$, then $\frac{2}{4}$ will not be accepted although the two fractions are numerically equal.
5. All other answers should be written in simplest form.
6. If a unit is indicated in the problem, the answer must be given in that unit. For instance, if the problem asks for the answer in hours, you cannot give your answer in minutes. Furthermore, you don't need to write the unit, as the graders will assume your answer is in the units asked for in the problem.
7. There is no penalty for guessing.
8. Ties will be broken based on the number of correct responses to the last ten questions. If a tie remains, then the correct responses to the last five questions will break the tie.
9. We will announce how much time is remaining often during the test.

2 Contest

1. Compute $9 \cdot (1.0000 + 0.10000 + 0.01000 + 0.00100 + 0.00010 + 0.00001) + .00001$.

2. At a tennis tournament, every player plays exactly one match against each of the other players. How many matches will be played if twenty people participate?

3. If $x + 2 \times x + 3 \times x = 6$, find x.

4. In how many ways can 7 be written as a sum of three (not necessarily distinct) positive whole numbers? The order of the numbers does not matter.

5. If the ratio of the circumference to the area of a circle is $1 : 3$, what is its radius?

6. Sherry and Jenny were going for a walk when they noticed that the rocks on the pathway formed a pattern. The first stone was light gray, the second stone was medium gray, the third stone was dark gray, the fourth stone was white, the fifth stone was light gray, etc. If the pattern continues, what color will the 333rd stone be?

7. In a zoo with thirty animals: some are rabbits and the rest are penguins, Hannah counts thirty-five pairs of legs. How many penguins are there?

8. A hamburger costs \$5.95 and a soda costs \$2.55. In cents, how much does Kun-soo pay to buy two hamburgers and two sodas?

9. If $a + b + c = 5$ and $2 \times a + 2 \times b + c = 8$, what is the value of c?

10. The Bergen County Academies math test has fifty problems and lasts one hour and thirty minutes. If Bob decides to spend the same amount of time on each problem, how many minutes will he spend on problem 42?

11. Evaluate $12 + 34 \times 56$.

12. David and Zuming are dividing thirty apples. David only wants a multiple of two apples while Zuming only wants a multiple of three apples. Each wants at least one apple. How many ways can they divide the apples amongst themselves so that no apples are left over?

13. What is the largest integer n such that n and n+13 are both perfect squares?

14. Kevin is stacking blocks in a pyramid. The first layer contains two blocks, the second layer contains four blocks, the third layer contains six blocks, and so on. If the pyramid has seventeen layers, how many blocks is it composed of?

15. What is $2 + 2 \times (2 + 2 \times (2 + 2 \times (2 + 2 \times 2)))$?

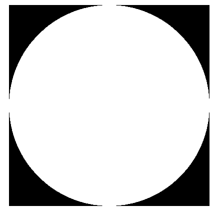
16. Many integers can be expressed as a sum of 3 perfect squares. For example, $5 = 2^2 + 1^2 + 0^2$. 7 is the smallest positive integer which cannot be expressed in this way. What is the next smallest integer that cannot be expressed as a sum of 3 perfect squares?

17. Anand bought a 72-foot long wire fence for his rectangular garden and wants to surround the biggest possible area. One side of the garden is adjacent to his house, so he only needs to surround three sides. In square feet, what is the biggest area he can surround?

18. Define a?b to be equal to $\dfrac{2 \cdot a + 3 \cdot b}{a - b}$. Find (3?4)?2.

19. If Starbursts are only sold in bags of 6, what is the least number of bags Austin must buy to get 49 Starbursts?

20. A circle is inscribed in a square with a side length of 2. What is the area of the square not inside the circle?



21. How many positive integers less than 80 have exactly four factors?

22. The angles A, B, C, and D ($A < B < C < D$) in a quadrilateral form an arithmetic series. What is $\dfrac{B+C}{A+D}$?

23. Alex Zhu goes to school for nine hours. When he gets home at 5:00 P.M., he does two hours of math. He then spends an hour eating, and does math until 1:00 A.M. He then goes to sleep for seven hours. If Alex does not multitask and can go to and from school in no time, what percent of the day does he spend doing math?

24. Evaluate
$$\frac{1}{\frac{1}{2}+\frac{1}{4}}$$

25. Hannah has only dimes and quarters in her coin purse. She notices that if she adds five pennies, eight nickels, and four dimes to her purse, she doubles the amount of money. What is the maximum number of coins with which she could have started out?

26. How many integers from 1 to 100 inclusive are multiples of 12 but not 15?

27. Janet has an unknown number of gumballs. If she gives one-third of her gumballs to Jim, then Janet and Jim will have the same number of gumballs. If Jim originally had 17 gumballs, how many gumballs did Janet originally have?

28. If $x^2y = 30$, find x when $y = 120$ and $x < 0$.

29. The sum of three consecutive even numbers is 30. What is the product of the first two numbers?

30. Evaluate $(-3) \cdot (-2) \cdot (-1) \cdot \dfrac{1}{36} \cdot (1) \cdot (2) \cdot (3)$.

31. Kelvin is preparing for the Intermediate Math Open. Each day, he does two more problems than he did the previous day. If he did one problem the first day, how many problems will he have done after the eleventh day?

32. What is the largest integer we cannot obtain by adding 3's and 7's?

33. The cost of supplying food for Tim's party was \$1000. If the cost of entrance to the party is \$5, how many people need to come for Tim to have \$500 left (after paying for the food) to buy presents for himself?

34. How many integers from 1 to 100 (inclusive) leave a remainder of 2 or 4 when divided by 5?

35. If one Jenny equals two Kristinas, three Kristinas equal six Lisas, and ten Lisas equal fifteen Margoes, how many Jennies equal 180 Margoes?

36. The area of a square is equal to its perimeter. What is its side length?

37. What is last digit of 2^{16}?

38. Ian is filling in a sequence with a specific pattern: 1, 2, 3, 6, 11, 20, 37, 68, a, b. Compute $a - b$.

39. Jordan is solving a calculus problem that requires him to give a single integer answer. Let p be the probability he gets the problem right and q be the probability he gets the problem wrong. Assuming that there is no partial credit, what is $p + q$?

40. Jordan buys 5 Häagen Dazs ice creams and 3 steaks for \$9. Bryan buys 13 Häagen Dazs ice creams and 15 steaks for \$27. If Fahmid buys one Häagen Dazs ice cream and one steak, how much does he pay?

41. Kevin's book has 707 pages. How many sixes will he come across in the page numbers?

42. If I have twenty-five meters of wire and want to use it to make a rectangle with the biggest possible area, what is the area of that rectangle in square meters?

43. Find $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{99 \cdot 100}$.

44. Kevin has two red balls, three blue balls, and one white ball. All balls of the same color are indistinguishable. How many ways are there of arranging the six balls in a line?

45. Evaluate $(-0)+(-1)-(-2)+(-3)-(-4)+(-5)-(-6)+(-7)-(-8)+(-9)-(-10)+...+(-99)-(-100)$.

46. Find the area of the triangle bounded by the x-axis, the y-axis, and the line $y = -7x + 14$.

47. A man is born in 1900 and dies in 2000. He is not 100 years old when he dies. How old is he?

48. Yi is trying to guess the locker combination to a lock. The lock code is a 3 digit number, with the digits from 0 to 9. He knows that the first digit is not a 9 and the second digit is not a 9. He also knows that the product of the digits is 0. What is the maximum possible sum of the digits of the locker code? (The first digit can be zero)

49. Bob is driving down a straight road in his car. He drives for two hours at 45 miles per hour and then for three more hours at 40 miles per hour. How far, in miles, does Bob travel in total?

50. If I reverse the digits in a two-digit number, the number increases by 18. What is the greatest possible sum of the new number and the old number?