Bergen County Academies Math Competition - 8th Grade

General Rules

- Calculators are not allowed.
- This is an individual test, so you may not communicate with anyone else taking it.
- Once time begins, we will not answer any questions about the problems.
- You will have 90 minutes to solve 50 problems. Once time is called, you must put down your pen or pencil and stop working.
- Scores will be posted on the website within a couple of days. Your score will appear next to your identification number.

Specifics

- You may use space on your test paper and additional scrap paper to do work. Your answers must be written on the answer sheet. We will not look at answers written on your test paper.
- Each problem has only one answer. If you put more than one answer for a problem, you will be marked wrong. When changing an answer, be sure to erase or cross out completely.
- Write legibly. If the graders cannot read your answer, it will be marked incorrect.
- Fractions should be written in lowest terms. For example, if the answer is $\frac{1}{2}$, then $\frac{2}{4}$ will not be accepted although the two fractions are numerically equal.
- All other answers should be written in simplest form.
- If a unit is indicated in the problem, the answer must be given in that unit. For instance, if the problem asks for the answer in hours, you cannot give your answer in minutes. Furthermore, you don't need to write the unit, as the graders will assume your answer is in the units asked for in the problem.
- There is no penalty for guessing.
- Ties will be broken based on the number of correct responses to the last ten questions. If a tie remains, then the correct responses to the last five questions will break the tie.
- We will announce how much time is remaining often during the test.

1. How many positive integers divide 23 evenly?

2. Jie, David, and Ben each have x buttons. Jie gives 12 buttons to David who now has 6 less than twice the number of buttons Jie has. Ben then gives y buttons to David who now has 20 buttons more than Ben has. Ben then gives y more buttons to Jie, who now has the same number of buttons as Ben. How many buttons does David have now?

3. Find $(-2010) \times (-2009) \times (-2008) \times \cdots \times 2008 \times 2009 \times 2010$.

4. In a standard 52-card deck, what is the probability of choosing a royal card (Jack, Queen, or King) of a red suit (Hearts or Diamonds)?

5. A, B, and C are three points in the plane, so that $\overline{AB} = 2$, $\overline{BC} = 3$, and $\overline{CA} = 5$. Let M be the midpoint of $\overline{AC}$. Find the distance from M to B.

6. Three tiles are marked H and two other tiles are marked A. The five tiles are randomly arranged in a row. What is the probability that the arrangement reads $HAHAH$?

7. Chan drives the first half of a 100 mile trip at 60 mph and then drives the second half at 30 mph. What is his average speed for the trip?

8. $log_3\ 27 =?$

9. Express 64 in base 6.

10. Convert 57 – a number written in base 10 – to base 3.

11. Find the sum of the first 20 terms of the arithmetic sequence 5, 10, 15, ...

12. Find the last digit of 3^{30}.

13. If x and y are real numbers such that$x^2 = -|y^2 - 1|$, what is $x^{2009} + y^{2010}$?

14. In regular hexagon $ABCDEF$, what is the ratio $\overline{AC} : \overline{AB}$?

15. In-Sung Na took the AIME, a 15 question test. Given that he got 14 of the 15 right and the chances of him getting any particular problem correct is the same, what is the probability he got none of the first 10 wrong?

16. What is the smallest number x such that $4x$ is a perfect cube and $5x$ is a perfect square?

17. For a two digit number ab, define $f(ab) = ba$. For example, $f(23) = 32$ and $f(10) = 01 = 1$. Find the greatest common factor of $f(10), f(11), \ldots, f(99)$.

18. Evaluate $\frac{i^{2010}}{i^{2008}} + \frac{i^{2009}}{i^{2008}}$, where $i = \sqrt{-1}$.

19. If the weather forecast says that there is a $\frac{3}{5}$ chance of rain on any given day, what is the probability it will rain at least once over the next four days?

20. If $4x^2 - 9y^2 = 15$ and $2x + 3y = 3$, find $2x - 3y$.

21. How many vertices does a regular octahedron have?

22. Find the number of solutions to $7x + y = 2010$, where x and y are positive integers.

23. What is the remainder when 2^{27} is divided by 9?

24. 4 yellow beads and 6 green ones are randomly arranged in a line. Given that the first and last bead are the same color, what is the probability first bead is green?

25. The time is now 2:28 PM. In an analog clock, the hour hand and the minute hand will make an angle of 180 degrees in x minutes, where x is positive and as small as possible. Find x.

26. The Master of Debates wins 90% of his debates against The Apprentice of Debates. Find the probability that he wins a best of three competition against The Apprentice of Debates.

27. Consider ten boxes each filled with ten Pokeballs. Ash finds that among them is one box containing defective Pokeballs which weigh one gram less than normal Pokeballs. Ash has a digital scale which can measure the mass of the Pokeballs, either one or many at a time. Find the least number of weighings with which Ash can find out the defective box.

28. If $x+\frac{1}{x}=1$, find $x^2+\frac{1}{x^2}$.

29. David Yang has four children with integer ages. If the product of their ages is 7, find the sum of their ages.

30. In a 10 by 10 square, the length is increased by 10% and the width is decreased by 10%. Find the percent change of the area of the square.

31. GOOGOL's stock starts at \$600 dollars per share. If the stock falls by 10% and than rises by 10% of the new price, what is the new stock price in dollars?

32. Find the smallest positive integer that can be written as the sum of four distinct positive perfect squares.

33. A boy wants to write all the numbers from 1 to 1000. On day 0, he starts counting from "0", and then writes 120 digits a day (so, on the first day, he'd count 65 numbers – 10 one-digit numbers and 55 two-digit numbers, for a total of 120 digits). On what day does he write one thousand?

34. Victoria has five shirts (white, blue, black, brown, red) and three pants (blue, black, brown) How many different outfits can she make if she does not want her shirt and pants to be the same color?

35. When $2.4\overline{81}$ is expressed as a fraction of integers, it yields $\frac{a}{b}$, where a and b are relatively prime. Find $a+b$.

36. How many digits are there in 32000011^2?

37. If $\frac{1}{x}+3=\frac{13}{x}$, find x.

38. 5 lines are drawn. What is the largest number of points of intersections of these lines?

39. If $\frac{59}{\sqrt{3}+\sqrt{5}+\sqrt{7}}$ can be expressed in the form $a\sqrt{3}+b\sqrt{5}+c\sqrt{7}-d\sqrt{105}$, where a, b, c, d are positive integers, find $a+b+c+d$.

40. The area of a square is numerically 9 times more than the square's perimeter. Find the side length of the square.

41. A quartic (a polynomial of fourth degree) passes through the points $(0,0)$, $(1,0)$, $(2,0)$, $(3,0)$, and $(4,1)$. What is the value of the polynomial at $x=5$?

42. Find the number of zeroes at the end of 2010!.

43. Find the radius of the circle passing through all three vertices of an equilateral triangle of side length 4.

44. Circles $\mathcal{C}_1$ and $\mathcal{C}_2$ have the same center, with $\mathcal{C}_2$ inside $\mathcal{C}_1$. A chord of length 12 in $\mathcal{C}_1$ is tangent to $\mathcal{C}_2$. Find the area of the region in $\mathcal{C}_1$ and not in $\mathcal{C}_2$.

45. While trying to calculate the sum $21+22+23+24+25+26+27+28+29$, Jongwhan forgets to take into account one plus sign, getting 2601 as his result. If he forgot the plus sign between $2a$ and $2b$ (where a and b are the ones digit of the two digit number), find $a+b$ where a and b are consecutive integers between 1 and 9 inclusive.

46. $2n^2-16$ is a prime number. Find all integer values for n.

47. The numbers $2^{\frac{1}{222}}, 3^{\frac{1}{333}}, 4^{\frac{1}{444}}$, and $5^{\frac{1}{555}}$ are arranged in an increasing order. The second and third number on this list are x and y. Find $\log_x y$.

48. x, y, z are real numbers satisfying $\frac{|x-y|}{2}+\sqrt{|2y+z|}+z^2-z+\frac{1}{4}=0$. Find $(y+z)^x$.

49. What is the smallest integer that 16,000 can be multiplied by to get a number with 10 zeros at the end?

50. A palindrome is a number that reads the same way forward or backward. For example, 987656789, 66, 9 are palindromes, but 1212 and 2010 are not. How many odd five-digit numbers that start with 5 are palindromes?