6th Grade Solutions

1. $4 \times 2.22222 + .00002 = 8.88888 + .00002 = \mathbf{8.8889}$

2. $1 + 1 \times (1 + 1 \times (3))) = 1 + 4 = \mathbf{5}$

3. Just plug in the numbers. $6\clubsuit(2\clubsuit3) = 6\clubsuit\left(\frac{2*2+3*3}{4*3-2*2}\right) = 6\clubsuit\left(\frac{13}{8}\right) = \frac{6*2+3*(13/8)}{4*(13/8)-2*6} = -\mathbf{\frac{135}{44}}$

4. This is the famed Fibonacci Sequence in which the previous two numbers add up to the next number. Continuing the sequence, the next numbers will be 13, 21, 34, and 55. **55** is the tenth number of the sequence.

5. 2, 3, 5, 7, 11, 13, 17, and 19 are the **8** prime numbers less than 20.

6. We count the number of multiples of 2, 3, and 6 less than 100. There are 49 multiples of 2, 33 multiples of 3, and 16 multiples of 6 less than 100. There are $49 + 33 - 16 = 66$ integers less than 100 that are multiples of 2 or 3. Hence, there are $99 - 66 = \mathbf{33}$ integers less than 100 that are not multiples of 2 or 3.

7. The formula for summation of integers from 1 to n is $\frac{n\times(n+1)}{2}$. Thus the answer is $\frac{26\times(27)}{2} = \mathbf{351}$.

8. If two sides have the same length, then at least two angles must be the same. Thus, if we assume 50 is the duplicate angle. The other angle must be either 80 or 50 since $180 = 50 + 50 + 80$. If we assume 50 degrees is the unique angle, then the duplicate angle must be 65 since $180 = 65 + 65 + 50$. Thus the sum of all distinct possible values is $80 + 50 + 65 = \mathbf{195}$.

9. First, turn feet into yards for consistency. So he climbs 1000 yards every day and falls 400 yards every night. So the net amount travelled in one day-night is 600 yards. If we divide 10800 by 600, we get 18 days. However, problems like this can be tricky, so we examine what happens on the second to last day, i.e. the 17th day. After 16 days he has climbed up 9600 feet, so on the 17th he will climb to 10600 feet, 200 short of the summit. So we are right and our answer is **18** days

10. If two triangles are similar, then by definition the ratio of all corresponding sides is constant. From the problem, we know that $\overline{AB}$ is similar to $\overline{DE}$, $\overline{BC}$ is similar to $\overline{EF}$. The ratio of $\overline{AB}$ to $\overline{DE}$ is 3 to 6, which is $\frac{1}{2}$. The ratio of $\overline{BC}$ to $\overline{EF}$ is 4 to some number. Since the ratio of be $\frac{1}{2}$, the number must be **8**.

11. Multiplication is commutative. So $20 \times 0.1 \times 0.2 = 20 \times 0.2 \times 0.1$. Therefore, $20 \times 0.1 \times 0.2 - 20 \times 0.2 \times 0.1 = \mathbf{0}$.

12. When you subtract 35, an odd number, from a number x to get a number y, either x or y will be odd and the other number will be even. Since the only even prime number is 2, y must be 2. Hence, $x = 2 + 35 = \mathbf{37}$.

13. The probability for a coin to land on heads is $\frac{1}{2}$. Therefore, for each flip, there is a $\frac{1}{2}$ chance of getting heads. The coin is flipped 6 times, which gives $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{2^6} = \mathbf{\frac{1}{64}}$.

14. Since all even numbers greater than 2 are divisible by 2, none of them can be a prime number. Investigating only the odd numbers greater than 90 and less than 100, we see that $91 = 7 \times 13$, $93 = 3 \times 31$, $95 = 5 \times 19$, and $99 = 9 \times 11$ are not prime numbers. However, **97** is not divisible by any number other than 1 and itself, so it must be the answer.

15. The number of days until they will next be working on the same day is the least common multiple of 5, 3, 6, and 7, which is $2 \cdot 3 \cdot 5 \cdot 7 = \mathbf{210}$.

16. If we group this sequence into groups of four we get $1+2-3+4 = 4, 5+6-7+8 = 12, ..., 97+98-99+100 = 196$. We can see that the pattern is for every 4 numbers in each group, we have to add 8 more than the previous. The sequence is $4 + 12 + 20 + ... + 196$. The formula to add numbers that are in an arithmetic sequence are $\frac{first+last}{2} \times Numbers\ you\ are\ adding$. In our case it would be $\frac{4+196}{2} \times 25 = 100 \times 25 = \mathbf{2500}$.

17. Obviously, 0 works. Factoring out x (since all other $x \neq 0$) gives us $x^2 = 1$. The only other numbers that work are ±1, for a total of **3** solutions.

18. $100 \geq c + w$ so $100 - c \geq w$. Substitute $100 - c$ for w in $c - \frac{w}{4}$. So $c - 25 + \frac{c}{4} \leq 57$. Then solve for c, and you get $c \leq 65.6$. If he answers 66 questions, regardless of how many he gets wrong, he won't get a score of 57. If he answers 65, then he can get 32 wrong and leave 3 blank to get 57. Thus the answer is **65**.

19. Isabel needs 10 binders, $\frac{10\times3}{2} = 15$ packs of paper, and 15 packs of pens. 10 binders is \$30, 15 packs of paper is \$30, and 15 packs of pens costs \$15, so it'll cost her \$**75** to go to school.

20. The sequence is a repetition of the four numbers 1, 9, 6, 3. Since 2010 leaves a remainder of 2 when divided by 4, the 2010th number of the sequence will be the second number in the sequence. This means it is **9**.

21. At Amazing Ice, there are $29 \times 15 \times 3 = 1305$ possible orders. At Country Creamery, there are $27 \times 19 \times 2 = 1026$ possible orders. Amazing Ice has more possible orders, with $1305 - 1026 = \mathbf{379}$ more possible orders.

22. First he has to cut along the wrapping paper. So he has to cut five feet. At a cutting speed of 3 in/sec, he can do this is 20 seconds. Next, he needs to cut another 12 inches for the square (since one side is shared with the edge of wrapping paper, you do not actually have to cut the whole 16 inches), so that will take 4 seconds. Finally, add 60 seconds for the tape. Thus, the answer is **84** seconds.

23. The factors of 20 are 1, 2, 4, 5, 10, 20. The reciprocal of a number is 1 over that number. So $\frac{1}{1} + \frac{1}{2} + \frac{1}{4} + \frac{1}{5} + \frac{1}{10} + \frac{1}{20} = \mathbf{\frac{21}{10}}$.

24. Arrange the given conditions into three equations and add them. This will result in two bags of each costing 262 cents. Divide by two to find the answer, **131**.

25. In 48 minutes, the fastest runner ran 8 laps, and the slowest one ran 6 laps, so the third person must have ran 7 laps. So in 48 minutes he ran 7 laps; therefore the answer is $\mathbf{\frac{48}{7}}$.

26. The minute hand moves 60 minutes or 360° every hour. Thus, each minute corresponds to 6 degrees. At the 9 minute mark, the minute hand will have traveled 54° from the 12 (0 minute mark). The hour hand moves 30° ($\frac{1}{12}$ of 360) every hour, or 60 minutes, so after 9 minutes, it will travel $\frac{9}{60} \times 30^{c}irc = 4.5°$ from the 6. The 6 is already 180° from the 12 mark, so it forms a 184.5° with the 12 (at least the larger of the two). Subtract 54 from 184.5, since they were formed from the same reference, which equals **130.5°**.

27. Heejin takes an hour (60 minutes) to run 5 miles. Jenny runs $\frac{1}{8}$ a mile per minute for the first 3 miles. So it takes her 24 minutes to run the first 3 miles. The last 2 miles, she runs at a pace of $\frac{1}{16}$ a mile per minute. So she takes 32 minutes to run the last two miles so she takes 56 minutes total. Thus **Jenny** wins.

28. $A + J = 210$, $J + M = 205$, $M + A = 221$. Adding all three and dividing by two, $A + J + M = 318$. $A + J + M - (M + A) = 318 - 221 = \mathbf{97}$.

29. Mike's number is equal to $16n + 9$, for some number n. So, Mike's number is also equal to $4(4n + 2) + 1$, and leaves a remainder of **1** when divided by 4. OR Take any number, for example 25, that when divided by 16 has a remainder of 9. 25 divided by 4 leaves a remainder of **1**.

30. The population of a colony with a beginning population of b bacteria after a time of h half-hours is $b \times 2^h$. So our answer is $2 \times 2^8 = 2^9 = \mathbf{512}$.

31. The volume of a $12 \times 15 \times 18$ box is $12 \times 15 \times 18 = 3240$, while the volume of each $3 \times 3 \times 3$ block is $3^3 = 27$. Then $\frac{3240}{27} = \mathbf{120}$ small blocks will fit in the box.

32. For every 60 miles the barbie moves, Kelly will move 5 miles. In three hours, the barbie will move $3 \times (60 - 5) = \mathbf{165}$ miles with respect to Kelly.

33. In the first portion of the race, Ben Llama runs at 6 mph for 30 minutes (a half-hour), covering a total distance of $6 \times \frac{1}{2} = 3$ miles. At this point there are 7 miles left in the race; if Ben walks all but the last half-mile, he will have walked 6.5 miles, which will take $\frac{6.5}{1} = 6.5$ hours. For the last 0.5 miles, Mr. Llama sprints at 10 mph, so the last portion of the race will take $\frac{0.5}{10} = 0.05$ hours, or 3 minutes. 30 minutes +(6.5 hours) × (60 minutes) +3 minutes = **423** minutes.

34. We have to use combinations because the order does not matter. We have to do ${}_8C_4$. ${}_8C_4 = \frac{8!}{4!\times4!} = \frac{8\times7\times6\times5}{4\times3\times2\times1}$ $= 7 \times 2 \times 5 = \mathbf{70}$.

35. $2x + 2y + 2z = 160$ and $2x + 3y + 4z = 180$, so $y + 2z = 20$ (from this, we have $z \leq 10$) $3x + 3y + 3z = 240$ and $2x + 3y + 4z = 180$, so $x = 60 + z$. Therefore, $100x + 80y + 50z = 100(60 + z) + 80(20 - 2z) + 50z = 7600 - 10z$. When $z = 0$, we get $M = 7600$ and when $z = 10$, we get $m = 7500$. Hence, $M - m = \mathbf{100}$.

36. The area of a triangle of is $\frac{1}{2}\times b \times h$. The base is length $\overline{BC}$ and the height is $\overline{AB}$. Thus, the area is $.5 \times 15 \times 10 = \mathbf{75}$.

37. Because the range of the scores is 6, one person must have scored a 6 and one person must have scored a 0. Furthermore, because the median of the scores is 0.5, the 4th lowest score and the 3rd lowest score are 1 and 0, respectively. Hence, the first and second lowest scores are 0 as well. We currently know the scores of everyone but the one who scored the fifth lowest score; these scores are 0, 0, 0, 1, and 6. Because the mean of the scores is 1.5, the final missing score must be 2, so our answer is **0,0,0,1,2,6**.

38. The number of k is equal to the number of perfect squares below 64. The perfect squares are $0, 1, 4, 9, 16, 25, 36, 49$. Thus the answer is **8**.

39. A 5×5 magic square uses the numbers 1 through 25. Let us consider all the horizontal rows. There are 5 rows, which in total contain all the numbers 1 through 25. Each of these rows contains the same sum. Thus, the total of all the sums of the horizontal rows is the sum of all the numbers 1 through 25, just arranged in some way. The sum of the numbers 1 through 25 is $1 + 2 + \cdots + 24 + 25$. We can pair the first and last element, the second and second to last element, etc, such that we get 12 pairs with a sum of 26 each, and a remaining 13. Thus the sum is $12 \times 26 + 13 = 325$. However, this is the total sum of all the rows. Each row will have a fifth of this sum (since each contributed equally). So each row is equal to 65, and so each row, column, and diagonal sums to **65**.

40. We can factor $x^2 - 4x + 3 = 0$ as $(x - 1)(x - 3) = 0$, or $x = 1, 3$. Plugging in either gives us **4** OR If $x^2 - 4x + 3 = 0$, we see that x cannot be 0. Thus, divide both sides by x to get $x - 4 + \frac{3}{x} = 0$, i.e. $x + \frac{3}{x} = 4$

41. Break this problem up into steps. First, calculate the number of boys by dividing the total number of kids (69) by 3; this results in 23 boys. Using this knowledge we can take the difference between the total number of kids (69) and number of boys (23) to get the number of girls (46). Since we know that 8 girls want vanilla, the rest must want chocolate. To find the number of girls that want chocolate, we take the difference of the total number of girls (46) and number of girls that want vanilla (8) to get 38 girls that want chocolate. Lastly, we take the number of kids that want chocolate (40) and subtract the number of girls that want chocolate (38) to get the number of boys that want chocolate: **2**.

42. We can rewrite $x^2 - 2x - 2$ as $x^2 - 2x + 1 - 3$, or $(x - 1)^2 - 3$. Since any square is nonnegative, the minimum is achieved when $(x - 1)^2 = 0$, or $x = 1$. Plugging this in give us **-3** as the minimum

43. Let c be the number of chickens and p the number of pigs. $C + P = 70$, $2C + 4P = 200$. If we multiply the first equation by 2 we get $2C + 2P = 140$. If we then subtract the second equation from this equation we get $2P = 60$, hence $P = 30$, so there are **30** pigs.

44. The first triangle we insert into a corner of the larger equilateral triangle. We are then left with a region which is a trapezoid with bases 0.99 and 1. An equilateral triangle with length 0.99 will be unable to cover the bottom base of 1, but will be able to cover nearly all of it. There is only a small region left to cover (parallelogram with side lengths 0.01) and this can easily be covered with another triangle. Therefore, the answer is **3**.

45. The sequence is generated according to the following pattern: $(n + 1)$th number$-(n)$th number $= n$. Hence, the answer is **2010**.

46. The formula for the surface area of a cube is $6 \times e^2$, where e is the length of the edge. $54 = 6 \times e^2$ means that $e = 3$. The radius is twice this, so $r = 2e = 6$. Therefore, the area of the circle is $\pi r^2 = \mathbf{36}\pi$.

47. The total amount of liters is 4 and the total amount of vinegar in the solution is $0.4 + 1.6 = 2$. This means the percent is $\frac{2}{4}\times 100 = \mathbf{50}\%$.

48. From the given, Robert can paint a fence per hour, whereas Mike can paint half a fence per hour. Therefore, if they work together, they can paint $1+\frac{1}{2} = \frac{3}{2}$ of a fence per hour, so it would take them $\frac{2}{3}$ hours to paint a fence. Since $\frac{2}{3}$ hours is equal to $\frac{2}{3}\times 60 = 40$ minutes, the answer is **40**.

49. If d is the smallest factor, $\frac{123{,}456}{d}$ must be the largest factor. We look for the smallest factor: 123456 is even, so 2 is a divisor. $\frac{123{,}456}{2}$=**61728**

50. Using guess and check, one finds that 2 and 6 work because $2 + 6 + 12 = 20$. So that means $x + y =$ **8**. OR Add 1 to both sides and factor the left side into $(x + 1)(y + 1) = 21$. The only values of x and y that work are 2 and 6. $2 + 6 =$ **8**.