

Bergen County Academies Math Competition - 4th Grade

General Rules

- Calculators are not allowed.
- This is an individual test, so you may not communicate with anyone else taking it.
- Once time begins, we will not answer any questions about the problems.
- You will have 90 minutes to solve 50 problems. Once time is called, you must put down your pen or pencil and stop working.
- Scores will be posted on the website within a couple of days. Your score will appear next to your identification number.

Specifics

- You may use space on your test paper and additional scrap paper to do work. Your answers must be written on the answer sheet. We will not look at answers written on your test paper.
- Each problem has only one answer. If you put more than one answer for a problem, you will be marked wrong. When changing an answer, be sure to erase or cross out completely.
- Write legibly. If the graders cannot read your answer, it will be marked incorrect.
- Fractions should be written in lowest terms. For example, if the answer is $\frac{1}{2}$, then $\frac{2}{4}$ will not be accepted although the two fractions are numerically equal.
- All other answers should be written in simplest form.
- If a unit is indicated in the problem, the answer must be given in that unit. For instance, if the problem asks for the answer in hours, you cannot give your answer in minutes. Furthermore, you don't need to write the unit, as the graders will assume your answer is in the units asked for in the problem.
- There is no penalty for guessing.
- Ties will be broken based on the number of correct responses to the last ten questions. If a tie remains, then the correct responses to the last five questions will break the tie.
- We will announce how much time is remaining often during the test.

1. Find $2 + 5 \times 6$.
2. Sue is standing on the 9th rung of a ladder. She goes up 6 rungs, down 2 rungs, up 3 rungs, and down 9 rungs. She then goes up 11 rungs and ends up at the top rung. How many rungs are on the ladder?
3. Compute $\frac{666666}{333333}$.
4. Compute $3 \times (48 - 2) + 3^2 \times 5 - 2$.
5. What is the average (arithmetic mean) of .4, .04, .004, and .0004?
6. In a cookie recipe, 5.5 cups of flour are needed to bake 2 loaves of bread. How many cups of flour are needed to bake 5 loaves of bread?
7. Chef J reaches into a chocolate storage bin, which houses gratuitous amounts of two types of chocolate: dark chocolate and white chocolate. If he blindly grabs pieces of chocolate from the storage bin, how many must he grab to ensure he has at least three pieces of the same type of chocolate?
8. Compute $1.354 + 0.79 + 2.005 + 1.8 + 4.05 + 0.001$.
9. How many different 4 digit numbers can be formed by rearranging the digits of 2011? (Numbers starting with 0, e.g., 0121 and 0211, do not count).
10. Jason has \$93 but he wants to have prime number of dollars. What is the least amount of money he needs to spend in order to have a prime number of dollars?
11. Find $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32}$.
12. Harry Potter is mixing potions in Potions class. He is supposed to add $2\frac{1}{2}$ cups of newt scales, $\frac{1}{8}$ cup of unicorn blood, $\frac{11}{4}$ cups of treacle tart, and $\frac{2}{3}$ cups of phoenix tears. Assuming he does this correctly, what will be the final volume of the potion?
13. Jessica the Big Fish is placed in a tank with dimensions 1 foot by 2 feet by 4 feet. If Jessica's body volume is 1 cubic foot, then what percent of the tank's volume is occupied by Jessica's body?
14. Compute 11^4 .
15. If $11x + 27 = 1234321$, find $22x + 54$.
16. A palindrome is a number that reads the same back and forth. For example, 12321 is a palindrome. Find the number of 6-digit palindromes (palindromes cannot start with the digit 0.)
17. Two variables are called *inversely proportional* if their product is constant. If x^2 and y are inversely proportional, and $y = 4$ when $x = 6$, find y when $x = 4$.
18. Find the number of solutions to the equation $|x + 5/2| = -216$.
19. It takes 3 rabbits 3 minutes to eat 3 carrots. How many minutes does it take for 6 rabbits to eat 6 carrots?

20. A drug company is taking a survey in which they need 1000 people to respond. How many people should they enlist to take the survey if they expect 60% of the survey-takers to drop out of the survey?
21. A set of n consecutive integers beginning with -15 sums to 16. What is n ?
22. One parasprite spawns a new parasprite every 20 minutes. If we start out with one parasprite, how many parasprites will there be after two hours?
23. Dumbledore is eating jelly beans. In a bag of jelly beans, there are 4 blue jelly beans, 9 red jelly beans, and 7 black jelly beans. If Dumbledore randomly picks three jelly beans and eats each one right after taking it out of the bag, then what is the probability that none of the three jelly beans he chose are blue?
24. Granny knits 20 stitches per minute, and one sweater is composed of 2100 stitches. How many hours does Granny take to knit 2 sweaters?
25. In how many ways can 4 people arrange themselves in a line?
26. Find all integers n for which all of the interior diagonals of a convex n -gon are of equal length.
27. Find the smallest positive integer larger than 1 that leaves a remainder of 1 upon division by each of 2, 3, 4, 5, and 6.
28. If $8^{3x+4} = 16^{5x}$, find x .
29. Given that x and y are real numbers such that $(2x + 3y - 5)^2 + (x - 2y + 7)^2 = 0$, find x .
30. If you roll three dice, what is the probability that the product of the numbers on the dice is odd?
31. Suppose a, b, c, d are distinct digits such that the three-digit numbers $7\overline{a}9$ and $2\overline{b}4$ sum to the four-digit number $\overline{cd}13$. Find all possible values of $a + b + c + d$.
32. If $33.33\dots\%$ of y is 20, what is the sum of the factors of y ?
33. A palindrome is a number that reads the same back and forth. Find the number of 8-digit palindromes whose digits sum to 15.
34. A hemisphere of radius 5 is glued to the top of a cylinder with radius 5 and height 10. Find the surface area of the resulting solid.
35. In how many ways can 6 indistinguishable algebra books, 4 indistinguishable combinatorics books, and 1 geometry book be arranged on a bookshelf?
36. In how many distinct ways can the letters in the word "COOPER" be arranged such that the first and last letters are vowels?
37. What day of the week will October 17, 2012 be?
38. What is the smallest positive integer n greater than 1 such that n^2 is a cube and n^3 is a square?
39. A snail is at the bottom of a long tube that measures 20 meters. On a given day, the snail will travel 3 meters up the tube, and during the night it will sink 1 meter down the tube. On what day will the snail finally reach the top of the long tube?

40. Find the 200th term in the following sequence: 1, 2, 2, 3, 3, 3, 4, 4, ...
41. Find the last digit of $9^{8^{7^{6^{5^{4^{3^{2^1}}}}}}}$.
42. Find $1.9\overline{8}$ (that is, 1.9888... with repeating 8's) in simplest fractional form.
43. Archimedes, Bernoulli, Cauchy, Descartes, and Euler are standing in a line. How many ways can the five line up if Descartes and Euler want to stand next to each other, and Archimedes wants to be at the front?
44. Find the smallest positive integer that cannot be represented as the sum of three not necessarily distinct perfect squares.
45. A regular fair six-sided die is rolled twice. What is the probability that the first number rolled divides the second number rolled?
46. Find all integers x such that $x^2 + 2x - 8$ is a prime number.
47. Find the number of zeroes at the end of $((3!)!)!$, where $n! = n \times (n - 1) \times \cdots \times 2 \times 1$.
48. How many integers n between 1 and 2011, inclusive, have the property that $n^2 + 2n + 3$ is divisible by 3?
49. Applejack, Twilight Sparkle, and Rainbow Dash are picking apples at Sweet Apple Acres. If it takes Applejack and Twilight Sparkle 3 days to pick all the apples, Applejack and Rainbow Dash 4 days to pick all the apples, and Twilight Sparkle and Rainbow Dash 6 days to pick all the apples, how long would it take all three of them together to pick apples?
50. Find the volume of the region defined by $3x + 4y + 5z \leq 60$ and $x, y, z \geq 0$.