Bergen County Academies Math Competition - 6th Grade

General Rules

- Calculators are not allowed.
- This is an individual test, so you may not communicate with anyone else taking it.
- Once time begins, we will not answer any questions about the problems.
- You will have 90 minutes to solve 50 problems. Once time is called, you must put down your pen or pencil and stop working.
- Scores will be posted on the website within a couple of days. Your score will appear next to your identification number.

Specifics

- You may use space on your test paper and additional scrap paper to do work. Your answers must be written on the answer sheet. We will not look at answers written on your test paper.
- Each problem has only one answer. If you put more than one answer for a problem, you will be marked wrong. When changing an answer, be sure to erase or cross out completely.
- Write legibly. If the graders cannot read your answer, it will be marked incorrect.
- Fractions should be written in lowest terms. For example, if the answer is $\frac{1}{2}$, then $\frac{2}{4}$ will not be accepted although the two fractions are numerically equal.
- All other answers should be written in simplest form.
- If a unit is indicated in the problem, the answer must be given in that unit. For instance, if the problem asks for the answer in hours, you cannot give your answer in minutes. Furthermore, you don't need to write the unit, as the graders will assume your answer is in the units asked for in the problem.
- There is no penalty for guessing.
- Ties will be broken based on the number of correct responses to the last ten questions. If a tie remains, then the correct responses to the last five questions will break the tie.
- We will announce how much time is remaining often during the test.

1. Sue is standing on the 9th rung of a ladder. She goes up 6 rungs, down 2 rungs, up 3 rungs, and down 9 rungs. She then goes up 11 rungs and ends up at the top rung. How many rungs are on the ladder?

2. Compute $\frac{666666}{333333}$.

3. Chef J reaches into a chocolate storage bin, which houses gratuitious amounts of two types of chocolate: dark chocolate and white chocolate. If he blindly grabs pieces of chocolate from the storage bin, how many must he grab to ensure he has at least three pieces of the same type of chocolate?

4. Compute $1.354 + 0.79 + 2.005 + 1.8 + 4.05 + 0.001$.

5. How many different 4 digit numbers can be formed by rearranging the digits of 2011? (Numbers starting with 0, e.g., 0121 and 0211, do not count).

6. Compute 11^4.

7. Two variables are called *inversely proportional* if their product is constant. If x^2 and y are inversely proportional, and $y = 4$ when $x = 6$, find y when $x = 4$.

8. A set of n consecutive integers beginning with -15 sums to 16. What is n?

9. In how many ways can 4 people arrange themselves in a line?

10. A baby chick walks forward 3 steps, then backward 1 step. This pattern continues until the chick immediately stops at a traffic light 200 forward-steps away from his starting position. How many backward-steps did the chick take before finally reaching the traffic light?

11. 1 frag is equal to 2 fregs, 3 fregs are equal to 4 frigs, and 5 frigs are equal to 6 frogs. How many frags equal 48 frogs?

12. Compute $2+4+6+8+10+12+14+16+18+20-1-3-5-7-9-11-13-15-17-19$.

13. Valentino is writing a three-word poem by randomly choosing (not necessarily distinct) words among the words"love," "is," "blind,"and "cake." What is the probability that his poem will read, "cake is cake"?

14. Rohil uses the spinner to select a number between 1 to 5 inclusive, each with equal probability. Michael uses a different spinner to select a number from 1 to 6 inclusive, each with equal probability. Compute the probability that Rohil selects an odd number and Michael selects a prime number.

15. Let $f(x) = x + 1$ and $g(x) = 2x$. Find $f(g(f(g(f(g(1))))))$.

16. Find the area of a triangle with side lengths 6, 8, and 10.

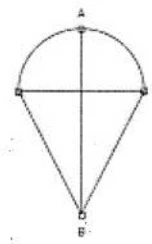
17. Find all integers n for which all of the interior diagonals of a convex n-gon are of equal length.

18. A palindrome is a number that reads the same back and forth. Find the number of 8-digit palindromes whose digits sum to 15.

19. In $\triangle ABC$, there is a point D on AB and a point E on BC such that $\angle AED = 40$, $\angle ADE = 80$, and $\angle DBC = 100$, find $\angle BCE$.

20. Given that x and y are real numbers such that $(2x + 3y - 5)^2 + (x - 2y + 7)^2 = 0$, find x.

21. Joe is coloring a picture of an ice cream cone, which consists of a semicircle placed atop an isosceles triangle, as shown in the diagram. The radius of the semicircle is 2 centimeters, and the area of the entire figure is $2\pi + 16$. If B is the vertex of the isosceles triangle and A is the point on the semicircle directly opposite B. Find the distance from A to B.



22. Suppose a, b, c, d are distinct digits such that the three-digit numbers $7\overline{a}9$ and $2\overline{b}4$ sum to the four-digit number $\overline{cd}13$. Find all possible values of $a + b + c + d$.

23. How many arrangements are there of the letters in the word "ANAGRAM"?

24. If $\dfrac{49^{27x}}{7^{9x}} = 49$, find x.

25. How many integer solutions are there to the equation $2x + 5y = 100$, where x and y are both larger than 5?

26. How many pairs of integers (a, b) are there such that $a^2 - b^2 = 36$?

27. Find all real x such that $x^3 + x^2 + x + 1 = 0$.

28. A snail is at the bottom of a long tube that measures 20 meters. On a given day, the snail will travel 3 meters up the tube, and during the night it will sink 1 meter down the tube. On what day will the snail finally reach the top of the long tube?

29. Find the 200th term in the following sequence: $1, 2, 2, 3, 3, 3, 4, 4, \ldots$

30. What is the smallest positive integer n greater than 1 such that n^2 is a cube and n^3 is a square?

31. Find the coefficient of x^2 in the expansion of $(x + \frac{1}{x})^{2011}$.

32. Find the last digit of $9^{8^{7^{6^{5^{4^{3^{2^{1}}}}}}}}$.

33. How many solutions are there to the equation $(x + 1)^{x^2+3x+2} = 1$?

34. Let Γ be a circle centered at O. Given that AB is a chord of length 12 in the circle, and the distance from O to AB is 3, compute the area of Γ.

35. If $x^3 - 3x^2 - 4x = 30$ and $x^3 - 6x^2 + 4x = -5$, find x.

36. Boris randomly selects a point inside the circumcircle of an equilateral triangle. Compute the probability that the point selected is outside the triangle but inside the circle.

37. Find the smallest positive integer that cannot be represented as the sum of three not necessarily distinct perfect squares.

38. Find the number of zeroes at the end of $((3!)!)!$, where $n! = n \times (n-1) \times \cdots \times 2 \times 1$.

39. Compute $1^2 - 2^2 + 3^2 - 4^2 + ... - 50^2 + 51^2$.

40. James has 200 feet of fence to build a rectangular enclosure around a house which has one side on the river. This means that the house needs only to be covered on three of its sides. What is the largest possible area that the fence can enclose?

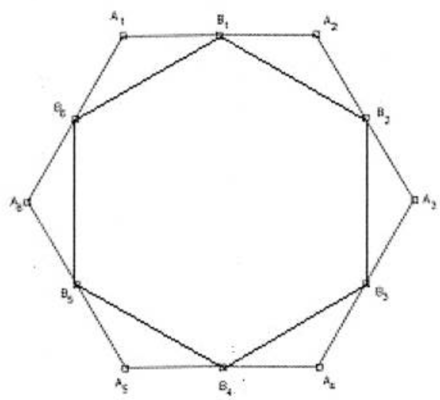
41. In $\triangle ABC$, $AB = 4$, $BC = 5$, $CA = 6$, and the bisector of angle A intersects BC at D. Find the length of BD.

42. Find the quadratic equation whose coefficient of x^2 is 1, and whose roots are the squares of the roots of $x^2 + 4x - 2$.

43. Let $\triangle ABC$ be an equilateral triangle of side length 4, and let A_1, B_1, C_1 be the midpoints of segments BC, CA, and AB, respectively. Let A_2, B_2, C_2 be the midpoints of segments B_1C_1, C_1A_1, and A_1B_1, respectively. Find the ratio of the area of $\triangle A_2B_2C_2$ to the area of $\triangle ABC$.

44. Archimedes, Bernoulli, Cauchy, Descartes, and Euler are standing in a line. How many ways can the five line up if Descartes and Euler want to stand next to each other, and Archimedes wants to be at the front?

45. Regular hexagon $A_1A_2A_3A_4A_5A_6$ has side length 1. Take B_i to be the midpoint of A_i and A_{i+1} for $i = 1, 2, 3, 4, 5, 6$, with $A_7 = A_1$. Find the ratio of the area of $B_1B_2B_3B_4B_5B_6$ to the area of $A_1A_2A_3A_4A_5A_6$.



46. Two real numbers between 0 and 1 are chosen randomly. Find the probability that their sum is less than $\frac{1}{2}$.

47. Given that $929 = 23^2 + 20^2$, and that $2 \cdot 929$ can be expressed in the form $c^2 + d^2$ for some positive integers c and d, find $c + d$.

48. If $x - \frac{1}{x} = 3$, find $x^2 + \frac{1}{x^2}$.

49. Find the sum of the digits of $(10^{100} - 1) - \frac{10^{100} - 1}{10^{20} - 1}$.

50. Let $A_1A_2A_3\dots A_{20000}$ be a regular polygon with 20,000 sides. If $A_1A_{10001} = 20$, find the integer closest to the area of $A_1A_2A_3\dots A_{20000}$.