Bergen County Academies Math Competition - 7th Grade

General Rules

- Calculators are not allowed.
- This is an individual test, so you may not communicate with anyone else taking it.
- Once time begins, we will not answer any questions about the problems.
- You will have 90 minutes to solve 50 problems. Once time is called, you must put down your pen or pencil and stop working.
- Scores will be posted on the website within a couple of days. Your score will appear next to your identification number.

Specifics

- You may use space on your test paper and additional scrap paper to do work. Your answers must be written on the answer sheet. We will not look at answers written on your test paper.
- Each problem has only one answer. If you put more than one answer for a problem, you will be marked wrong. When changing an answer, be sure to erase or cross out completely.
- Write legibly. If the graders cannot read your answer, it will be marked incorrect.
- Fractions should be written in lowest terms. For example, if the answer is $\frac{1}{2}$, then $\frac{2}{4}$ will not be accepted although the two fractions are numerically equal.
- All other answers should be written in simplest form.
- If a unit is indicated in the problem, the answer must be given in that unit. For instance, if the problem asks for the answer in hours, you cannot give your answer in minutes. Furthermore, you don't need to write the unit, as the graders will assume your answer is in the units asked for in the problem.
- There is no penalty for guessing.
- Ties will be broken based on the number of correct responses to the last ten questions. If a tie remains, then the correct responses to the last five questions will break the tie.
- We will announce how much time is remaining often during the test.

1. Sue is standing on the 9th rung of a ladder. She goes up 6 rungs, down 2 rungs, up 3 rungs, and down 9 rungs. She then goes up 11 rungs and ends up at the top rung. How many rungs are on the ladder?

2. Compute $\dfrac{666666}{333333}$.

3. What is the remainder when 12345678 is divided by 3?

4. If $11x + 27 = 1234321$, find $22x + 54$.

5. Compute $2 + 2 + 4 + 8 + 16 + 32 + 64 + 128 + 256$.

6. A palindrome is a number that reads the same back and forth. For example, 12321 is a palindrome. Find the number of 6-digit palindromes (palindromes cannot start with the digit 0.)

7. It takes 3 rabbits 3 minutes to eat 3 carrots. How many minutes does it take for 6 rabbits to eat 6 carrots?

8. Find the value of $1 + 3 + 5 + 7 + \cdots + 45$.

9. Groovy-Band sold $\dfrac{11}{12}$ of the available tickets for their latest show, and total ticket sales were \$93,500. What would have been the total ticket sales, in dollars, if Groovy-Band had sold all of the available tickets?

10. Dumbledore is eating jelly beans. In a bag of jelly beans, there are 4 blue jelly beans, 9 red jelly beans, and 7 black jelly beans. If Dumbledore randomly picks three jelly beans and eats each one right after taking it out of the bag, then what is the probability that none of the three jelly beans he chose are blue?

11. Let $f(x) = x + 1$ and $g(x) = 2x$. Find $f(g(f(g(f(g(1))))))$.

12. Find the sum of the prime factors of 2012.

13. A baby chick walks forward 3 steps, then backward 1 step. This pattern continues until the chick immediately stops at a traffic light 200 forward-steps away from his starting position. How many backward-steps did the chick take before finally reaching the traffic light?

14. One parasprite spawns a new parasprite every 20 minutes. If we start out with one parasprite, how many parasprites will there be after two hours?

15. Two variables are called *inversely proportional* if their product is constant. If x^2 and y are inversely proportional, and $y = 4$ when $x = 6$, find y when $x = 4$.

16. If $\dfrac{49^{27x}}{7^{9x}} = 49$, find x.

17. Given that x and y are real numbers such that $(2x + 3y - 5)^2 + (x - 2y + 7)^2 = 0$, find x.

18. A hemisphere of radius 5 is glued to the top of a cylinder with radius 5 and height 10. Find the surface area of the resulting solid.

19. Find the third smallest integer with exactly 3 divisors.

20. Find all integers n for which all of the interior diagonals of a convex n-gon are of equal length.

21. How many integer solutions are there to the equation $2x + 5y = 100$, where x and y are both larger than 5?

22. Find the coefficient of x^2 in the expansion of $(x + \frac{1}{x})^{2011}$.

23. Find the last digit of $9^{8^{7^{6^{5^{4^{3^{2^{1}}}}}}}}$.

24. What day of the week will October 17, 2012 be?

25. How many solutions are there to the equation $(x+1)^{x^2+3x+2} = 1$?

26. If $x^3 - 3x^2 - 4x = 30$ and $x^3 - 6x^2 + 4x = -5$, find x.

27. Let Γ be a circle centered at O. Given that AB is a chord of length 12 in the circle, and the distance from O to AB is 3, compute the area of Γ.

28. Find the smallest positive integer that leaves a remainder of 2 when divided by 4, a remainder of 3 when divided by 5, and a remainder of 4 when divided by 6.

29. A regular fair six-sided die is rolled twice. What is the probability that the first number rolled divides the second number rolled?

30. Let r, s, t be the roots of the cubic $x^3 - 6x^2 + 5x + 1$. Find $(2-r)(2-s)(2-t)$.

31. Find the number of zeroes at the end of $((3!)!)!$, where $n! = n \times (n-1) \times \cdots \times 2 \times 1$.

32. In $\triangle ABC$, $AB = 4$, $BC = 5$, $CA = 6$, and the bisector of angle A intersects BC at D. Find the length of BD.

33. Compute $1^2 - 2^2 + 3^2 - 4^2 + \ldots - 50^2 + 51^2$.

34. A rectangle is inscribed in a circle. If the length and width of the rectangle are 24 and 10, respectively, what is the area of the circle?

35. Let $\triangle ABC$ be an equilateral triangle of side length 4, and let A_1, B_1, C_1 be the midpoints of segments BC, CA, and AB, respectively. Let A_2, B_2, C_2 be the midpoints of segments B_1C_1, C_1A_1, and A_1B_1, respectively. Find the ratio of the area of $\triangle A_2B_2C_2$ to the area of $\triangle ABC$.

36. Archimedes, Bernoulli, Cauchy, Descartes, and Euler are standing in a line. How many ways can the five line up if Descartes and Euler want to stand next to each other, and Archimedes wants to be at the front?

37. James has 200 feet of fence to build a rectangular enclosure around a house which has one side on the river. This means that the house needs only to be covered on three of its sides. What is the largest possible area that the fence can enclose?

38. Find all integers x such that $x^2 + 2x - 8$ is a prime number.

39. Find the quadratic equation whose coefficient of x^2 is 1, and whose roots are the squares of the roots of $x^2 + 4x - 2$.

40. Steven randomly draws two cards from a fair deck of 52 cards. If he picks a 6 and a 7, what is the probability that after he picks up another card, the sum of the values of his cards does not exceed 21? (Aces count as 1, and all face cards count as 10).

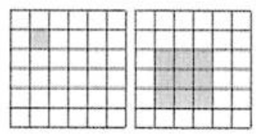
41. Two real numbers between 0 and 1 are chosen randomly. Find the probability that their sum is less than $\frac{1}{2}$.

42. Applejack, Twilight Sparkle, and Rainbow Dash are picking apples at Sweet Apple Acres. If it takes Applejack and Twilight Sparkle 3 days to pick all the apples, Applejack and Rainbow Dash 4 days to pick all the apples, and Twilight Sparkle and Rainbow Dash 6 days to pick all teh apples, how long would it take all three of them together to pick apples?

43. How many integers n between 1 and 2011, inclusive, have the property that $n^2 + 2n + 3$ is divisible by 3?

44. Find $11_2 + 11_3 + 11_4 + \cdots + 11_{100}$. ($11_b$ is taken to mean 11 in base b.)

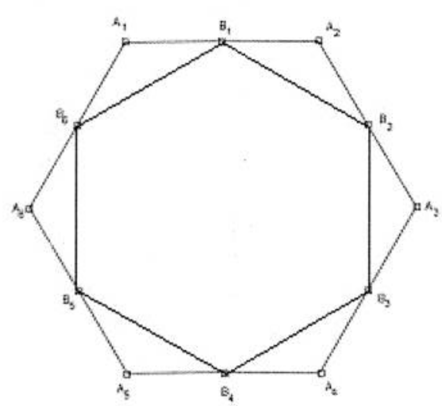
45. Find the number of squares that can be formed by the 1×1 squares on a 6×6 chessboard. (The diagram shows examples of some squares that can be formed.)



46. There are 5 distinct balls. If you pick two balls at random and Bob picks three balls at random, what is the probability that one of your balls will be among one of Bob's balls?

47. If $x - \frac{1}{x} = 3$, find $x^2 + \frac{1}{x^2}$.

48. Regular hexagon $A_1A_2A_3A_4A_5A_6$ has side length 1. Take B_i to be the midpoint of A_i and A_{i+1} for $i = 1, 2, 3, 4, 5, 6$, with $A_7 = A_1$. Find the ratio of the area of $B_1B_2B_3B_4B_5B_6$ to the area of $A_1A_2A_3A_4A_5A_6$.



49. Find the sum of the digits of $(10^{100} - 1) - \frac{10^{100} - 1}{10^{20} - 1}$.

50. Let $A_1A_2A_3 \ldots A_{20000}$ be a regular polygon with $20,000$ sides. If $A_1A_{10001} = 20$, find the integer closest to the area of $A_1A_2A_3 \ldots A_{20000}$.