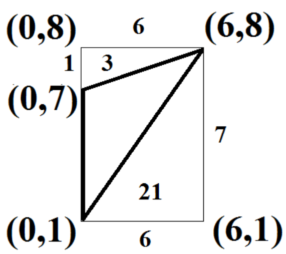
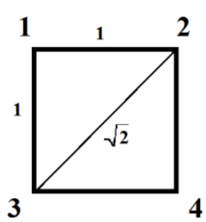
1. There are $3 + 4 + 7 + 3 + 2 + 4 + 8 = \boxed{\mathbf{31}}$ letters
2. James and Franklin can both take either Japanese or English, so each of them has two choices. So, there are $2 \cdot 2 = \boxed{\mathbf{4}}$ combinations.
3. $10 \cdot 20 = 200$ and $8 \cdot 30 = 240$. Therefore, there are $200 + 240 = \boxed{\mathbf{440}}$bags of cat food.
4. There are 12 inches in a foot, so Danny is $5 \cdot 12 + 3 = 63$ inches tall. He wants to be $75 + 2 = 77$ inches tall. He must to grow $77 - 63 = \boxed{\mathbf{14}}$ inches more.
5. There are a total of $3 + 4 + 2 = 9$ marbles. The probability of drawing a red marble is $\frac{3\text{ red marbles}}{9\text{ marbles}}$, which, when simplified, is $\boxed{\mathbf{\frac{1}{3}}}$.
6. $1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 = \boxed{\mathbf{255}}$.
7. The 9th grade hibernates for a total of $9 \cdot 100 = 900$ days. The 10th grade hibernates for a total $10 \cdot 100 = 1000$ days. The 11th grade hibernates for a total of $11 \cdot 100 = 1100$ days and the 12th grade hibernates for a total of $12 \cdot 100 = 1200$ days. Therefore, they all hibernate for a total of $900 + 1000 + 1100 + 1200 = \boxed{\mathbf{4200}}$ days.
8. Let the bigger number be x and the smaller y. We know that $x + y = 25$ and $x - y = 11$. Subtracting the first from the second, we get $2y = 14$, or $y = \boxed{\mathbf{7}}$.
9. If paint normally dries from 9:00 AM to 3:00 PM, it takes 6 hours to completely dry. With Betty watching, it should only take $\frac{6}{1.5} = 4$ hours. 4 hours from 9:00 is $\boxed{\mathbf{1:00P.M.}}$
10. The mean is $\frac{(1+1+3+4+4+5+6+6+6)}{9} = 4$. The median is also4 and the mode is 6. Their sum is $4 + 4 + 6 = \boxed{\mathbf{14}}$.
11. Half of all multiples of 3 are not multiples of 6. We can see there are 30 two-digit multiples of 3 $(12, 15 ... 96, 99)$, so $\frac{30}{2} = \boxed{\mathbf{15}}$ multiples of 3 are not multiples of 6.
12. Set $4x + 9 = x$ and solve to get $x = \boxed{\mathbf{-3}}$.
13. By the divisibility rule for 3, $8 + X + 5 + 4$ must be a multiple of 3. The numbers that make this work are 1, 4, and 7, and of those, the largest value is $X = \boxed{7}$.
14. The car traveled for $10 + 60 = 70$ seconds, so the car travels a distance of $5\frac{\text{m}}{\text{s}} \cdot 70\text{s} = 350\text{m}$. The train travels for a minute or 60 seconds. So it travels a distance of $10\frac{\text{m}}{\text{s}} \cdot 60\text{s} = 600$ m. Therefore the total distance between them initially is $350 + 600 = \boxed{\mathbf{950}}$m.
15. There are be $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ (or 5!) ways to arrange five different books in a row, because there are five choices for the first place on the shelf, then four choices for the second place, and so on. However, because there are 2 identical red books and 3 identical blue books, we need to divide by 2! and 3!, to eliminate where we have overcounted (there are 2! ways to order two identical red books in a line, but we only want to count one of those instances. The same is true for the blue books). This gives us $\frac{5!}{2! \cdot 3!} = \boxed{\mathbf{10}}$ possible arrangements.

16. If Mr. Lemma has x students, and since he has no students left over after putting them into both a square and 6 equal rows, x must be both a square number and divisible by 6. The only number between 100 and 150 that fits both of these criteria is $\boxed{\mathbf{144}}$.
17. The triangle is within a rectangle of size $7 \cdot 6$, and you can find the area of the triangle by subtracting the areas of the other triangles, so the area is $42 - 21 - 3 = \boxed{\mathbf{18}}$. Alternatively, since $Area = \frac{1}{2} b \cdot h$, take the segment from $(0,1)$ to $(0,7)$ as the base (with length 6) and drop the height from $(6,8)$ (which has length 6, as shown below). The area is therefore $\frac{1}{2} \cdot 6 \cdot 6 = \boxed{18}$.

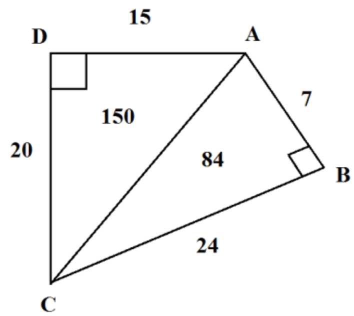


18. For the Robzhanians, it is 7:57 every 8 hours, and for people on Earth it is 7:57 every 12 hours. Therefore, it will be 7:57 again in the least common multiple of 8 and 12 hours, which is $\boxed{\mathbf{24}}$.
19. Note that we can fold an octagon symmetrically in two ways---either through the midpoints of two opposite sides or through two opposite vertices. Each has four ways, since there are four pairs of opposite sides and four pairs of opposite vertices. Thus the answer is $4 + 4 = \boxed{\mathbf{8}}$
20. Normal runners run $1 + 1 + 1 = 3$ units around the square, while Robin runs $1 + \sqrt{2} + 1 = 2 + \sqrt{2}$ units. Therefore, Robin runs $2 + \sqrt{2} - 3 = \boxed{\sqrt{\mathbf{2}} - \mathbf{1}}$ units more.



21. The mean, or average, of the 4 numbers is $\frac{3+7+5+2x}{4}$, which is also x, so cross-multiplying gives $3 + 7 + 5 + 2x = 4x$, and $\boxed{\frac{\mathbf{15}}{\mathbf{2}}} = x$.

22. The quadrilateral ABCD can be cut into two right triangles ADC and ABC, which have areas 150 and 84, respectively, so the total area is $150 + 84 = \boxed{\mathbf{234}}$.



23. $\frac{n+4}{n}$ is an integer and n is an integer. Therefore, for some integer i, the solution to $\frac{n+4}{n} = i$, or $n = \frac{4}{i-1}$, must be an integer. $\frac{4}{i-1}$ is an integer when $i = -3, -1, 0, 2, 3, 5$, so all possible values of n are $n = -1, -2, -4, 4, 2, 1$, and the sum is $-1 + -2 + -4 + 4 + 2 + 1 = \boxed{\mathbf{0}}$.

24. Paula's rate of painting is $\frac{1\text{ house}}{60\text{ minutes}}$, and the combined rate of Paula and Abhiram is $\left(\frac{1\text{ house}}{60\text{ minute}} + \text{Abhiram's rate}\right) = \frac{1\text{ house}}{40\text{ minutes}}$. Therefore, Abhiram's rate is $\frac{1\text{ house}}{40\text{ minutes}} - \frac{1\text{ house}}{60\text{ minutes}} = \frac{1\text{ house}}{120\text{ minutes}}$, so it will take Abhiram $\boxed{\mathbf{120}}$ minutes alone to paint one house.

25. Split the question up into whether there are 2,1, or 0 semitrucks present. If there are 0 semitrucks present, there can be 0,1,2,3,4, or 5 SUVs present, with the rest of the space being taken up by sedans. If there is 1 semitruck present, there can be 0,1,2, or 3 SUVs present, with the rest of the space being taken up by sedans. If there are 2 semitrucks present, there can be 0 or 1 SUV present, with the rest of the space being taken up by sedans. So, there are $6 + 4 + 2 = \boxed{\mathbf{12}}$ ways to arrange the sedans, SUVs, and semitrucks.

26. 90% of the time, Kipachu will do 100 damage, and 10% of the time, Kipachu will do 200 damage. Therefore, the average damage will be $100 \times 90\% + 200 \times 10\% = \boxed{\mathbf{110}}$ damage.

27. We will break this up into two cases. First, consider the probability of rolling at least one 2 or 5. By complementary probability, the probability of *not* rolling either a 2 or a 5 is $\frac{4}{6} \cdot \frac{4}{6} = \frac{16}{36} = \frac{4}{9}$. Thus, the probability of rolling a 2 or a 5 is $1 - \frac{4}{9} = \frac{5}{9}$. However, we must also subtract the two cases when we roll *both* a 2 and a 5, which happens with probability $\frac{2}{36} = \frac{1}{18}$. Therefore, the probability of rolling a 2 or a 5, but not both, is $\frac{5}{9} - \frac{1}{18} = \boxed{\mathbf{\frac{1}{2}}}$.

28. 0.108108108 ...can be written as $0.\overline{108}$. Let $x = 0.\overline{108}$. Then, $1000x = 108.\overline{108}$. Therefore, $1000x - x = 108.\overline{108} - 0.\overline{108} = 108$. So, $999x = 108$, or $x = \frac{108}{999}$, which, when simplified, is $\frac{4}{37}$, so $4 + 37 = \boxed{\mathbf{41}}$.

29. We know, from 1 across and 1 down, that a two-digit perfect cube and a two-digit perfect square must have the same tens digit. The only options for the tens digit are therefore 2 (27 and 25) or 6 (64 and 65). However, if the tens digit is 6, then by 3 across we must have a multiple of 13 that begins with a 4. However, no such multiple exists, so the tens digit cannot be 6. Instead, it must be 2. We are now looking for a two-digit multiple of 13 that begins with a 5, and a two-digit multiple of 8 that begins with 7. We see that putting 2 in the last two boxes, giving 72 and 52, satisfies the criteria. Our final crossword looks like this:

1 **2**	2 **7**
3 **5**	**2**

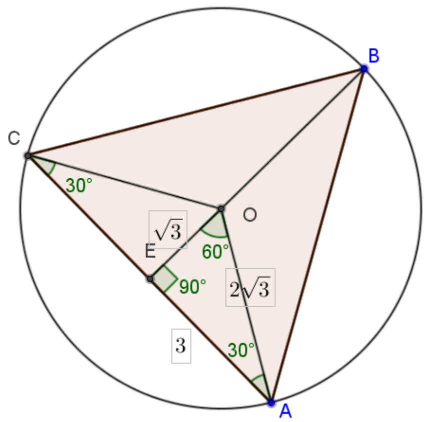
30. See picture.
31. See picture.
32. See picture.
33. We are looking for numbers not divisible by 2 or 3, since these are the only prime factors of 48. There are $\frac{48}{2} = 24$ multiples of 2 and $\frac{48}{3} = 16$ multiples of 3 that are at most 48, giving us a sum of 40 multiples of 2 or 3. However, we over-counted multiples of 6 twice, since they are both divisible by 2 and 3, so we must subtract the number of multiples of 6 from our count, which is $\frac{48}{6} = 8$. Hence there are $40 - 8 = 32$ multiples of 2 or 3 that are at most 48, and therefore there are $48 = 32 = \boxed{\mathbf{16}}$ numbers that are relatively prime to 48.
34. Since $\overline{DE}$ is parallel to $\overline{AB}$, $\Delta CED \sim \Delta CBA$ by AA similarity. This also means that the ratio of the areasof the two triangles is the square of the ratio of their sides so $[CDE] = \frac{1}{9}[CBA]$. Since $[CBA] = 171, [CDE] = 19$. Therefore the area of quadrilateral $DEAB$is $[CBA] - [CDE] = 171 - 19 = \boxed{\mathbf{152}}$.
35. We want to compute $[DEG] + [GFC]$ so we use the most basic formula for the area of a triangle: $\frac{bh}{2}$. We know the bases of the triangles are 3 and the height is 8 because they are both inscribed in the original rectangle. Therefore the area of one of the triangles is $\frac{3 \cdot 8}{2} = 12$ and the area of both of them is 24. The area of the rectangle is simply $6 \cdot 8 = 48$. The ratio of this combined area to the area of the rectangle is $\frac{24}{48} = \boxed{\frac{\mathbf{1}}{\mathbf{2}}}$.
36. Let the smaller number be $ABCD$. Then the digits of the larger number are $A + 1, B + 1, C + 1, D + 1$. Therefore, the larger number is 1111 greater than the smaller number. Let the smaller number be x. Then $x + (x + 1111) = 5823$. Solving for x, we have $x = 2356$. Therefore, the larger number is $2356 + 1111 = \boxed{\mathbf{3467}}$.
37. Let the rate of the ball be r and the time it takes it to hit Matt be t. Therefore the time it takes to travel back to Andrew from Matt is $8 - t$. Since Andrew starts running right when the ball was thrown, the total distance between him and Matt is $6 \cdot 8 + 20 = 68$m so the ball has to cover 20m at its normal speed and 48m at double its speed. Now we

can set up two equations: $\frac{20}{r} = t$ and $\frac{48}{2r} = 8 - t \rightarrow 20 = rt$and $24 = 8r - rt$.

Substituting rt into the second equation we get $24 = 8r - 20 \rightarrow r = \boxed{\frac{\mathbf{11}}{\mathbf{2}}}\frac{\text{m}}{\text{s}}$

38. The formula for the surface area of a sphere is $4\pi r^2$. Therefore the surface area of this sphere is $4 * 10 * 10 * \pi = \boxed{\mathbf{400\pi}}$.
39. Let this 3-digit number be abc. Then the equation we can write is $a^2 + b^2 + c^2 = 2(a + b + c)$. By subtracting $2a + 2b + 2c$ from both sides, and adding 3 to both sides, we can group terms to get: $(a - 1)^2 + (b - 1)^2 + (c - 1)^2 = 3$. The only way for 3 squares to sum to 3 is $1 + 1 + 1$. Therefore, $(a - 1)^2 = 1$. Since none of the digits are 0, we have that $a = 2$. Similarly, we obtain that $a = b = c = 2$. Therefore, our number is $\boxed{\mathbf{222}}$.
40. There are two options: either Andrew chooses two boys, or he chooses one boy and one girl. For the former option, he has $\frac{5\times 4}{2} = 10$ ways to choose two members for his band, while for the latter option, he has $5 \times 4 = 20$ ways to choose two members. Therefore, he has $20 + 10 = \boxed{\mathbf{30}}$ different pairs of people to choose from.
41. We first count the number of paths to get to $(3,3)$ first and multiply by the number of paths from $(3,3)$ to $(7,11)$. Since the ant can only go north or east, we can see that the ant must go north 3 times and east 3 times, taking 6 total steps to reach $(3,3)$. We can think of each north step as N and each east step as E so a path may look like NNNEEE depending on which the ant takes at each point. We can then see that the total number of paths to get to $(3,3)$ is just the total orderings of this string which is just $\binom{6}{3} = \frac{6\cdot 5\cdot 4}{3!} = \frac{120}{6} = 20$. Similarly, we need to walk 4 steps north and 8 steps east to go from $(3,3)$to $(7,11)$, which means we have a string looking like NNNNEEEEEEEE. This means that the total number of paths is $\binom{12}{4} = \frac{12\cdot 11\cdot 10\cdot 9}{4!} = \frac{11880}{24} = 495$. Therefore, we have the total number of paths for the whole walk is $495 \cdot 20 = \boxed{\mathbf{9900}}$ paths.
42. Let the numbers be $2k - 4, 2k - 2, 2k, 2k + 1, 2k + 4$. Multiplying all numbers gives us $(2k - 4)(2k - 2)(2k)(2k + 1)(2k + 4) = 32(k - 2)(k - 1)(k)(k + 1)(k + 2)$. Thus 32 must divide this number. Furthermore, there must be at least one integer out of the five consecutive integers $k - 2, k - 1, k, k + 1, k + 2$ that is divisible by 2, at least one divisible by 3, at least one divisible by 4 and at least one divisible by 5 (for example, take 1,2,3,4,5). Therefore the largest number that must always be a factor of this is $32 \times 2 \times 3 \times 4 \times 5 = \boxed{\mathbf{3840}}$.

    We can see that this it cannot be any larger since $2 \times 4 \times 6 \times 8 \times 10 = 3840$.



43. Let the circle be labeled O, and the triangle be labeled ABC. Notice that since this is an equilateral triangle, the center of O is the center of ΔABC. Therefore we have to find the distance from one of the points of the triangle to the center of the triangle to find the radius of the circle. By drawing the height from the center to one of the sides, we can see that the two right triangles formed are 30-60-90 triangles with the radius as the hypotenuse and the side opposite the 60 degree angle as3. Therefore, the hypotenuse, which is the radius, is simply $2 \cdot \left(\frac{3}{\sqrt{3}}\right) = \frac{6}{\sqrt{3}} = 2\sqrt{3}$. The circumference is then $2\pi r = \boxed{\mathbf{4\sqrt{3}\pi}}$.
44. If each person within a gender were to shake hands with each other person within the gender once, the total number of handshakes would be $\binom{n}{2}$. This is because the first person shakes hands with $n - 1$ other people, the second person would shake hands with the $n - 2$ people he did not shake hands with yet and so on reaching a total of $1 + 2 + \cdots + n - 2 + n - 1$. Since each male shakes twice, and each female shakes three times, we simply have to multiply this total by 2 and 3 for the males and females respectively. Therefore the total number of handshakes just within genders is $2 \cdot \binom{8}{2} + 3 \cdot \binom{12}{2} = 2 \cdot 28 + 3 \cdot 66 = 254$. For the opposite gender handshakes, we see that each male shakes hands with 8 other people so we just have to multiply $12 \cdot 8 = 96$ to get the number of handshakes here. Therefore the total number of handshakes is $254 + 96 = \boxed{\mathbf{350}}$.
45. The only ways for Bob to win are if he receives three votes right away or he receives three votes and Alice receives one vote in any order. The probability of him receiving three votes right away is $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$. The probability of him receiving three votes and Alice receiving one vote is the number of ways to arrange the ordering of these votes multiplied by $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{16}$. The ordering can be ordered $\binom{4}{1} = 4$ ways, but the ordering of the votes as Bob, Bob, Bob, then Alice cannot work, because the fourth turn will never happen. Therefore, there are only 3 valid orderings, so there is a $3 \cdot \frac{1}{16} = \frac{3}{16}$

chance of a 4-turn win.Therefore the total probability that Bob wins the election is $\frac{1}{8}+\frac{3}{16}=\boxed{\mathbf{\frac{5}{16}}}$.

46. We begin by subtracting the first equation from the third equation to get $x+2z=2$. Then we subtract the second equation from the third equation to get $-4x+2z=1$. Now we subtract the two equations above to get $5x=1\rightarrow x=\frac{1}{5}$. Then by substituting x into one of the equations above, we get $z=\frac{9}{10}$. Finally, by substituting x and z into one of the given equations, we get $y=0$. Therefore, the ordered pair is $\boxed{\left(\mathbf{\frac{1}{5}},\mathbf{0},\mathbf{\frac{9}{10}}\right)}$

47. Let $EF=x$. Therefore, $CG=12-2x$ and $BF=12-x$. We can see that triangles CEG and EBF are similar through AA similarity and therefore we have $\frac{EG}{BF}=\frac{CG}{EF}\rightarrow\frac{x}{12-x}=\frac{12-2x}{2x}\rightarrow 2x^2=(12-2x)(12-x)=144-36x+2x^2\rightarrow 36x=144\rightarrow$ $x=4$. Therefore $EG=4$ and $EF=8$. Finally to get the area of triangle AEC, we draw the perpendicular from E to AC and see that the area of the triangle is just $\frac{12\cdot(12-EF)}{2}=\frac{12\cdot 4}{2}=\boxed{\mathbf{24}}$.

48. We want to find the distance so we need to find the point of intersection between the line perpendicular to $y=x$ that passes through $(3,4)$ and the line $y=x$. The line perpendicular to $y=x$ has a slope of -1 and therefore this line has the form $y=-x+b$. This line also goes through $(3,4)$ so we can plug it in to get $4=-3+b\rightarrow b=7$. We now know the two lines and the intersection is the solution to this system of equations. Substituting $y=x$ into the second equation, we have $y=-y+7\rightarrow y=\frac{7}{2}$ and therefore $x=\frac{7}{2}$. Now we use the distance formula to find the distance from $\left(\frac{7}{2},\frac{7}{2}\right)$ to $(3,4)$: $\sqrt{\left(4-\frac{7}{2}\right)^2+\left(3-\frac{7}{2}\right)^2}=\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2}=\sqrt{\frac{1}{2}}=\boxed{\mathbf{\frac{\sqrt{2}}{2}}}$.

49. We can rewrite the equation as $2a(4+b)=3(12+b)$, or $a=\frac{3(12+b)}{2(4+b)}$. Since b must be a positive integer, the number with the lowest denominator is $\frac{3(13)}{2(5)}$, or 3.9. Because $\frac{(12+b)}{(4+b)}$ decreases as b increases, we know that a must be a positive integer less than or equal to 3.9 (1, 2, or 3) if b isa positive integer. The solution to $a=1=\frac{3(12+b)}{2(4+b)}$ is $b=-28$, which is not a positive integer. The solution to $a=2=\frac{3(12+b)}{2(4+b)}$ is $b=20$, so $(2,20)$ satisfies the conditions. The solution to $a=3=\frac{3(12+b)}{2(4+b)}$is $b=4$, so $(3,4)$ is also a pair that satisfies the conditions. Therefore, $\boxed{\mathbf{2}}$ ordered pairs satisfy the equation.

50. Since each beaker must have at least one chunk of potassium hydroxide, we have to find the ways to arrange four indistinguishable chunks of potassium hydroxide into 3 distinguishable beakers. We can see that the ways to arrange this are

$(4,0,0)$, $(3,1,0)$, $(2,2,0)$, and $(2,1,1)$ and any arrangement of these arrangements as the beakers are indistinguishable. For the first arrangement $(4,0,0)$, we have three ways to do it: $(4,0,0)$, $(0,4,0)$, $(0,0,4)$. For the second arrangement, we have six ways to do it: $(3,1,0)$, $(3,0,1)$, $(1,3,0)$, $(1,0,3)$, $(0,3,1)$, or $(0,1,3)$. For the third arrangement, we have three ways to do it: $(2,2,0)$, $(2,0,2)$,or $(0,2,2)$. For the last arrangement we also have three ways to do it: $(2,1,1)$, $(1,2,1)$, or $(1,1,2)$. This gives us a total of $3+6+3+3=\boxed{\mathbf{15}}$ total arrangements of the potassium chunks.