

1. How many words are in this question?
2. Define the operation  $*(a, b, c)$  as  $*(a, b, c) = \frac{a^2}{b} + \frac{b^2}{c} + \frac{c^2}{a}$ . Compute  $*(3, 1, 2)$ .
3. Let  $S$  be the number of sides a square has, and let  $P$  be the number of vertices a pentagon has. What is  $S + P$ ?
4. Calculate  $1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1}}}$ .
5. 25% of a number is 12. What is 37.5% of the same number?
6. How many prime numbers lie between 40 and 50 exclusively?
7. A quality McDonald's charges \$4.69 for a box and 10 McNuggets, and \$4.99 for a box and 20 McNuggets. How much does a box cost at McDonald's? Give your answer to the nearest hundredth of a dollar.
8. John thinks of a number, adds 10, divides the sum by 5, and then subtracts 3 from the quotient. If his resulting number is 0, what number did he start with?
9. What is the largest number of points at which two circles of different radii can intersect?
10. How many solutions are there to the equation  $|8x + 1| = 17$ ? Note that  $|a|$  denotes the distance from 0 to  $a$  on the number line.
11. A triangle with integer side lengths has two side lengths of 5 and 7. What is the difference between the smallest and largest possible values for the third side length?
12. Let  $n$  be the answer to this question. What is the value of  $3n + 12$ ?
13. If  $x, y \neq 0$ , find the value of  $\frac{(x \cdot y + y \cdot x)^2}{x^2 \cdot y^2}$ .
14. What is the smallest positive number that is both three times a square and twice a cube?
15. When rolling two fair standard dice at the same time, what is the probability of rolling a sum of a composite number?
16. Compute  $12^3 - 3 \cdot 12^2 + 3 \cdot 12 - 1$ .
17. Some of the problem writers made the following statements:
  - Kelvin the Frog: Alex ate the cake.
  - The Great Sabeenee: Steven is not lying.
  - Alex the Kat: I did not eat the cake.
  - Steven the Alpaca: AJ did not eat the cake.
  - AJ the Dennis: Kelvin ate the cake.If exactly one of these people is lying, who ate the cake?
18. There were originally 48 jellybeans in a jar. Josephine ate  $\frac{1}{4}$  of the jellybeans, she spilled  $\frac{2}{3}$  of the remaining jellybeans, then gave the rest to Jared. How many will Jared get to eat?
19. A square and an equilateral triangle have equal perimeters. What is the ratio of the area of the triangle to the area of the square?
20. The wheels on the bus go round and round. If their radius is 3 ft, how many rotations do they make during a trip of  $150\pi$  ft?

21. Air conditioner A can cool a room by 2 degrees per minute. Air conditioner B can cool a room by 3 degrees every two minutes. How long, in minutes, will it take for a room initially at 85 degrees to reach 30 degrees if both air conditioners are turned on?
22. A sheet of paper measures 3 feet by 5 feet. What is the maximum number of 4 inch by 6 inch cards that can be placed on this sheet of paper without overlapping or cutting the cards?
23. Two cars are driving on a highway. The first car is 60 miles behind the second car, and travels at 60 mph. After the first car has traveled 60 miles, the second car has moved 40 miles. If the cars remain at constant speeds, how much time from the start, in hours, does it take the first car to catch up to the second car?
24. The first term of a geometric sequence is 81, and the fourth term is 24. What is the sixth term?
25. The numbers from 1 to 25 will be arranged in a line such that the sum of each two adjacent numbers is a perfect square. What is the smallest number that can be adjacent to 18?
26. The distance from City  $A$  to City  $C$  is 100 miles. However, if you stop at City  $B$  on the way, the total distance from  $A$  to  $B$  to  $C$  is 260 miles. If City  $B$  is equidistant from  $A$  and  $C$ , how far off from the line  $AC$  is  $B$ , in miles?
27. At a school, 38 people swim, 39 people play tennis, and 9 people do neither. If there are 57 people in the school, how many people play both sports?
28. Kevin wins every game of the card game Egyptian Rat Screw when he plays with his eyes open. He wins  $\frac{5}{8}$  of his games when he plays with his eyes closed. If he plays  $\frac{1}{3}$  of his games with his eyes closed, what is the probability that he wins any game?
29. Josh wants to be like Albert Einstein. He heard Einstein got a B (83 – 87) in Math and decides to do the same. If he has a 95% for the year so far and only has a final exam left which is worth 20% of his grade, what is the highest grade he could get on the final and still get a B for the year?
30. A floor consists of  $1 \times 1$  unit tiles arranged in a square. The two diagonals intersect a total of exactly 2013 tiles. How many tiles are on the floor?
31. A  $10 \times 10 \times 10$  cube is painted red and then cut into 1000  $1 \times 1 \times 1$  cubes. How many of these smaller cubes are painted on exactly 2 faces?
32. What is the least positive integer  $n$  such that  $n \cdot 8!$  is a perfect square? Note that

$$n! = n \cdot (n - 1) \cdots 2 \cdot 1.$$

33. Rhombus  $ABCD$  has sides of length 5, with  $\angle A > 90^\circ$ . The altitude from  $A$  to  $CD$  intersects  $CD$  at  $E$ . Given that  $AE = 2EC$ , what is the length of  $DE$ ?
34. Beginning from 1, the pages of a book are written in sequential order. If exactly 2013 digits are written, how many pages are in the book?
35. A sequence is defined recursively as  $a_n = 3a_{n-1} - 2a_{n-2}$  for positive integers  $n \geq 2$ . If  $a_0 = 2$  and  $a_1 = 3$ , compute  $a_{10}$ .
36. How many squares of any size can be formed in a  $9 \times 9$  grid?
37. What is the value of  $\left(1 + \frac{1}{1}\right) \left(1 + \frac{1}{2}\right) \left(1 + \frac{1}{3}\right) \cdots \left(1 + \frac{1}{2013}\right)$ ?
38. A rectangle with side lengths of 6 and 8 is centered at the origin of a coordinate plane. This rectangle slowly spins around the origin, creating a circle. Compute the area of this circle.

39. Arthur, Wang, James, and Dennis had a race. Arthur didn't finish first or last, and James finished before Dennis. In how many different ways could the race have ended, provided that there were no ties?
40. A number is "almost-prime" if it has exactly 1 or 3 positive factors. How many numbers less than 2013 are almost-prime?
41. The numbers from 1 to 9 are arranged in a  $3 \times 3$  grid. The sum of each column, row, and diagonal is calculated. These eight sums add up to 124. What is the sum of the diagonals?
42.  $15! = 13A7674368000$ , for some digit  $A$ . Compute  $A$ .
43. A bug on a number line randomly chooses to move 1 unit left or right once every minute. What is the probability that after 6 minutes the bug will return to its original position?
44. Circles  $O$  and  $P$  are congruent circles with radius 3 and intersect at points  $A$  and  $B$ . If  $AB = 3$ , what is the area of the intersection?
45. A triangular number  $T_n$  is a number in the form  $1 + 2 + 3 + \cdots + n$  for some positive integer  $n$ . The first few triangular numbers are 1, 3, 6, 10, and 15. Compute  $\sqrt{T_{2012} + T_{2013}}$ .
46. In a box with red, blue, green, and orange marbles, all but 15 of the marbles are red, all but 20 of the marbles are blue, all but 25 of the marbles are green, and all but 27 of the marbles are orange. How many red marbles are in the box?
47. Kelvin the Frog and Ryan have an intense coin flipping contest. They first randomly choose a prime number between 1 and 1000. They then add 7 to this number. After this, they square the resulting number, find the sum of the digits of the new number, subtract 13, square this difference, and multiply by 2. They then construct a square with this side length, and throw a dart inside the square. They find the shortest length from this dart to a vertex, take the nearest integer, and then flip that many coins. The winner is the person with the greater number of heads (if there is a tie, they repeat the process). What is the probability that Kelvin the Frog wins the face-off?
48. James is buying cookies. If he can choose chocolate chip, sugar, or peanut butter cookies, how many different ways can he buy 10 cookies?
49. In  $\triangle ABC$ ,  $AB = 6$ ,  $BC = 8$ , and  $CA = 7$ . The angle bisector of  $\angle ABC$  intersects line  $AC$  at point  $D$ , and the angle bisector of  $\angle BDC$  intersects line  $BC$  at point  $E$ . Find  $CE$ .
50. Find the maximum number of elements that can be chosen from the set  $\{1, 2, 3, \dots, 2013\}$  such that the sum of any two chosen elements is not divisible by 3.