1. Express the expression $5 \div 4 \times 3 \div 2 \times 1$ as a fraction in lowest terms.
2. Define the operation $*(a, b, c)$ as $*(a, b, c) = \frac{a^2}{b} + \frac{b^2}{c} + \frac{c^2}{a}$. Compute $*(4, 3, 2)$.
3. Let S be the number of sides a square has, and let P be the number of vertices a pentagon has. What is $S + P$?
4. How many prime numbers lie between 110 and 120 exclusively?
5. What is the largest number of points at which two circles of different radii can intersect?
6. John has five different pairs of shorts, three pairs of shoes, and two shirts. If each outfit requires exactly one pair of shorts, one pair of shoes, and one shirt, how many different outfits can he make?
7. How many real solutions are there to the equation $|8x + 1| = 17$? Note that $|a|$ denotes the distance from 0 to a on the number line.
8. A square's perimeter is equal to its area. What is its side length, given that it is positive?
9. If 3 slaps are equal to 4 sleps, 8 sleps are equal to 5 slips, and 9 slips are equal to 13 slops, how many slops are equal to 54 slaps?
10. A bit is a unit of information that can take a value of 0 or 1. However, when a computer sends a bit, there is a $\frac{1}{3}$ chance that it malfunctions and sends the opposite value (e.g. sending a 1 when it should send a 0 or vice versa). To combat this, computer A sends the same bit 3 times to a receiving computer B, which changes the 3 bits into a single bit based on which number it receives more. For example, computer B would change 001 into 0. What is the probability that computer A successfully sends a bit to computer B?
11. Shawn has an m by n rectangular piece of paper, such that $\frac{m}{n} = \frac{1}{7}$. He then cuts the paper in half, such that the new rectangles each measure m by p. What is the value of $\frac{n}{p}$?
12. Let $i_0 = -1$ and $i_n = (-1)^{i_{n-1}}$ for all positive integers n. Compute $i_0 + i_1 + i_2 + \cdots + i_{2013}$.
13. When rolling 2 fair standard dice at the same time, what is the probability of rolling a sum of a composite number?
14. Compute $12^3 - 3 \cdot 12^2 + 3 \cdot 12 - 1$.
15. Let $t_0 = 5$ and $t_n = 5^{t_{n-1}}$. Find the last two digits of t_{2013}.
16. October 13th, 2013 is a Sunday. What is the next year when October 13th will be a Sunday?
17. Some of the problem writers made the following statements:
    - Kelvin the Frog: Alex ate the cake.
    - The Great Sabeenee: Steven is not lying.
    - Alex the Kat: I did not eat the cake.
    - Steven the Alpaca: AJ did not eat the cake.
    - AJ the Dennis: Kelvin ate the cake.

    If exactly one of these people is lying, who ate the cake?
18. Let $!n!$ be the product of all the numbers between n and $-n$ inclusive. Compute the remainder when $!2013!$ is divided by 2017.

19. Distinct integers a and b satisfy the equation $(a^x)(b^x) = (a+b)^x$ for some real number x. Compute x.

20. If $3x^2 + 1 = 45$, what is the value of $9x^4 + 6x^2 - 11$?

21. The numbers from 1 to n are arranged evenly around a circle in order. The numbers 17 and 38 are opposite to each other. Compute n.

22. Izzy has 17 distinct pairs of socks in her bag. If she takes them out one at a time at random, what is the smallest number of socks she needs to take out to guarantee that she took out two pairs of matching socks?

23. What is $(\sqrt{1 + 17\sqrt{16 \cdot 14 + 1}})^{\frac{1}{4}}$?

24. The distance from City A to City C is 100 miles. However, if you stop at City B on the way, the total distance from A to B to C is 260 miles. If City B is equidistant from A and C, how far off from the line AC is B, in miles?

25. What is the least positive integer n such that $n \cdot 8!$ is a perfect square? Note that

$$n! = n \cdot (n-1) \cdots 2 \cdot 1.$$

26. At a school, 38 people swim, 39 people play tennis, and 9 people do neither. If there are 57 people in the school, how many people play both sports?

27. $\triangle ABC$ is a right triangle with a right angle at C. Extend line AC to D such that $\angle ABD$ is a right angle. If $AC = 5$ and $BC = 12$, find the value of CD.

28. Kelvin can cut a piece of log into 6 pieces in 20 minutes. At the same rate, how long will Kelvin take to cut the log into 10 pieces, in minutes?

29. In $\triangle ABC$, let X be a point on BC such that $AX = BX = AC$. If $\angle BAX = 20°$, compute $\angle XAC$.

30. Sam drops a rock in a cylindrical container of radius 2cm half-full of water, placed on a flat surface. If the water rises 2cm, what is the volume of the rock, in cm^3?

31. Beginning from 1, the pages of a book are written in sequential order. If exactly 2013 digits are written, how many pages are in the book?

32. What is the value of $\left(1+\frac{1}{1}\right)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\cdots\left(1+\frac{1}{2013}\right)$?

33. 3 distinct numbers are chosen from the set $\{1, 1, 2, 3, 5, 8, 13, 21, 34, 55\}$. What is the probability that the 3 numbers form the side lengths of a non-degenerate triangle?

34. A sequence is defined recursively as $a_n = 3a_{n-1} - 2a_{n-2}$ for positive integers $n \geq 2$. If $a_0 = 2$ and $a_1 = 3$, compute a_{10}.

35. The length of the longest diagonal in a rectangular prism is 13. If all side lengths are integers, find the volume of the prism.

36. If a and b are distinct and nonzero real numbers such that $\frac{a^2+b^2}{a^2-b^2} = 2$, what is the value of $\frac{a^4+b^4}{a^4-b^4}$?

37. In a box with red, blue, green, and orange marbles, all but 15 of the marbles are red, all but 20 of the marbles are blue, all but 25 of the marbles are green, and all but 27 of the marbles are orange. How many red marbles are in the box?

38. An ant is standing on a cube of length 1. If the ant is standing on a vertex, what is the minimal distance it must travel on the cube to get to the opposite vertex of the cube?

39. Find the sum of all 4-digit numbers that each use the digits $1, 2, 3$, and 4 exactly once.

40. A bug on a number line randomly chooses to move 1 unit left or right once every minute. What is the probability that after 6 minutes the bug will return to its original position?

41. Let x be a value for which $x^2 - x + 1 = 0$. Compute the number a such that $x^4 + ax^2 + 1 = 0$.

42. If a and b are positive integers with $a > b$, such that $ab + a^2 = 144$ and $ab + b^2 = 112$, then compute the value of $a - b$.

43. Define $n+!$ to be $n+! = n! + (n-1)! + (n-2)! + \ldots + 2! + 1!$. Compute the last 2 digits of $2013+!$.

44. A diagonal is drawn from opposite endpoints of a 72×108 array of unit squares. How many unit squares contain any non-zero part of the segment inside them (meeting at a corner does not count)?

45. In equiangular hexagon $ABCDEF, BC = DE = FA = 2AB = 2CD = 2EF$. If the area of the hexagon is $52\sqrt{3}$, what is the perimeter of the hexagon?

46. Distinct a and b are chosen from the set $\{1, 2, \ldots, 2013\}$. What is the probability that the equation $x^2 - 2ax + b^2 = 0$ has both real solutions?

47. A certain chess match is 6 games long. A chess game can result in one player scoring 1 point and the other scoring 0, or both players scoring half a point. In how many ways can the match end up as a tie (each player scoring 3 points)?

48. A set of integers is bounded by the rules:

    - If $x \in S, x + 5 \in S$.
    - If $x \in S, (x + 7) \in S$.
    - $0 \in S$, but $1, 2, 3, 4 \notin S$.

    What is the largest integer not in any such set?

49. A set is "good" if it does not contain any 2 consecutive integers. How many subsets of the set $\{1, 2, 3, \ldots, 10\}$ are "good"?

50. A basketball initially has a radius of 6 inches. The ball is slowly deflated until the solid enclosed by the basketball is a perfect hemisphere. Compute the ratio of the volume of this hemisphere to the volume of the sphere.