1. $c = \frac{9b}{12} = \frac{3}{4}b = \frac{3}{4} \cdot \frac{2a}{3} = \frac{a}{2} = \frac{12}{2} = \boxed{6}$.

2. $*(3,1,2) = \frac{3^2}{1} + \frac{1^2}{2} + \frac{2^2}{3} = 9 + \frac{1}{2} + \frac{4}{3} = \boxed{\frac{65}{6}}$.

3. We see that this is equal to $1.1^3 = (1+.1)^3$. Using the fact that $(x+1)^3 = x^3+3x^2+3x+1$ and replacing x with .1, we find that $1.1^3 = \boxed{1.331}$.

4. $|8x+1| = 17$ means that either $8x+1 = 17$ or $8x+1 = -17$. Finding the solutions to both of these, $x = 2$ or $\frac{-9}{4}$. These are both perfectly valid, so there are $\boxed{2}$ solutions.

5. Let the square have side length s. Since the area of a square is equal to s^2 and its perimeter is equal to $4s$, the question tells us that $s^2 = 4s$. Since $s \neq 0$, we get that $s = \boxed{4}$.

6. This sum can be rewritten as $4(1+2+3+4+\cdots+25)$. We then recall the formula
$$1+2+\cdots+n = \frac{n(n+1)}{2}.$$
So, because $1+2+3+4+\cdots+25 = \frac{(26)(25)}{2} = 325$, our desired answer is $4 \times 325 = \boxed{1300}$.

7. Since the letters that the crowd chant repeat every three letters, every third position will be "A". 2013 is divisible by 3, so the 2013^{th} position will be $\boxed{A}$.

8. Let F be the number of fish that AJ gives to Soonho. After the transfer, AJ has $42 - F$ fish and Soonho has $3 + F$ fish. Since AJ has exactly double the amount of fish that Soonho has, $42 - F = 2(3+F)$. Solving this equation for F gives $F = \boxed{12}$.

9. We observe that for any x,
$$(x-1)^3 = (x^2-2x+1)(x-1) = x^3 - 3x^2 + 3x - 1.$$
(This is an example of the **Binomial Theorem**.) So, plugging in $x = 12$ yields
$$12^3 - 3 \cdot 12^2 + 3 \cdot 12 - 1 = (12-1)^3 = 11^3 = \boxed{1331}.$$

10. We see that there are 365 days in a non-leap year, and that 365 leaves a remainder of 1 when divided by 7, which is the number of days in a week. Thus, after each non-leap year, we see that October 13th will move ahead by one day. (For instance, in 2014 it will be a Monday.) However, on a leap year, there are 366 days, which leaves a remainder of 2 when divided by 7. So this tells us that in a leap year, October 13th will move ahead by *two* days. Since the year 2016 is a leap year, we see that in 2015 it will be a Tuesday; in 2016, Thursday; in 2017, Friday; in 2018, Saturday; in 2019, Sunday. Thus the answer is $\boxed{2019}$.

11. If x denotes the amount of money Kelvin had in the beginning, then we get that $x+10 = 3(x-10)$. Solving this equation yields $x = \boxed{\$20}$.

12. Let the radii of the solids be r and the height of the solids be h. Recall that the volume of a cone is $\frac{1}{3} \cdot \pi r^2 \cdot h$, while the volume of a cylinder is $\pi r^2 \cdot h$. Thus, the cylinder can hold $\frac{\pi r^2 \cdot h}{\frac{1}{3} \cdot \pi r^2 \cdot h} = \boxed{3}$ times as much water as the cone.

13. Let the common ratio of the geometric sequence be x. Thus, since the first term is 81 and the fourth term is 24, we have $81x^3 = 24$ which can be simplified to $x^3 = \frac{8}{27}$. Thus, $x = \frac{2}{3}$, and so the sixth term is $81 \cdot \left(\frac{2}{3}\right)^5 = \boxed{\frac{32}{3}}$.

14. The dimensions of the sheet of paper can be converted to 36 inches by 60 inches. So, if both the sheet of paper and the cards are positioned vertically, we can completely cover the paper by placing 6 cards across and 15 cards down. Therefore, our desired answer is $6 \times 15 = \boxed{90}$.

15. Since only one person is lying, Sabeenee must be telling the truth (Steven is not lying) as if he were lying, Steven is lying and then two people are lying. So that means Steven is not lying either, and therefore AJ did not eat the cake. Kelvin claims Alex ate the cake. If he is telling the truth, then Alex and AJ must be lying. So Kelvin must be telling a lie, and therefore, $\boxed{\text{Kelvin}}$ ate the cake.

16. Let the probability that the product of the numbers is even be p. Then $1 - p$ is the probability that the product is odd. The product of 3 integers can be odd only if each of the 3 integers are also odd. The probability of each integer being odd is $\frac{1007}{2014} = \frac{1}{2}$, so $1-p = (\frac{1}{2})^3 = \frac{1}{8}$. Therefore $p = \boxed{\frac{7}{8}}$.

17. Let the perimeter of the square and equilateral triangle be $12x$. Thus, each side of the square has length $3x$, while each side of the equilateral triangle has length $4x$. Since the area of a square with side length s is s^2, the area of the square is thus $9x^2$. Also, since the area of an equilateral triangle with side length s is equal to $\frac{s^2\sqrt{3}}{4}$, the area of the equilateral triangle is $\frac{(4x)^2\sqrt{3}}{4} = 4x^2\sqrt{3}$. The ratio of the area of the triangle to the square is thus $\frac{4x^2\sqrt{3}}{9x^2} = \boxed{\frac{4\sqrt{3}}{9}}$.

18. Suppose a and b are diametrically opposite such that $a < b$. Then there are equal amounts of numbers from a to b on the circle and from b to a. Since there are $n-2$ total numbers excluding a and b, there must be $\frac{n-2}{2}$ numbers from a to b. And since the numbers are placed in order, we have that
$$b - a = \frac{n-2}{2} + 1 = \frac{n}{2}.$$
Specifically, since $a = 17$ and $b = 38$, we see that $b - a = 38 - 17 = 21 = \frac{n}{2}$, and so $n = \boxed{42}$.

19. Note that $x = \sqrt{6 + \sqrt{6 + \sqrt{6 + x}}} = \sqrt{6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + x}}}}}} = \sqrt{6 + \sqrt{6 + \sqrt{6 + \ldots}}}$.
This can go on forever. However, if it goes on forever, then we can say that $x = \sqrt{6 + x}$. Squaring both sides and rearranging yields that $x^2 - x - 6 = 0$ so x is either 3 or -2. Clearly, x is positive, so $x = \boxed{3}$.

20. Note that the maximum sum of any two numbers from 1 to 25 is just $25+24 = 49$. Thus, the sum of any two adjacent numbers must be 1, 4, 9, 16, 25, 36, or 49. So, the pair of adjacent numbers including 18 must sum to either 25, 36, or 49. However, they can't sum to 36 as $36 = 18 + 18$ and a number can't be repeated. Also, they can't sum to 49 as $49 = 18 + 31$ and 31 doesn't fall between 1 and 25. However, $25 = 18 + 7$, and so the only number that can be next to 18 is $\boxed{7}$.

21. Let's say Soonho holds up all ten fingers x times. Then the other $10 - x$ times, he can hold at most 9 fingers. So in total, he can hold up at most $(10)(x) + (9)(10 - x)$ fingers, which is equal to $90 + x$. So since we want to maximize x, clearly $x = \boxed{5}$ does the trick.

22. The left-hand side can be written as $5(5^n) = 5^{n+1}$. Therefore, $n + 1 = 2013$ so $n = \boxed{2012}$.

23. If he picked the fair coin, there is a $\frac{1}{16}$ chance of it being heads 4 times. If he chose the unfair one, then there is a 1 chance that it would happen. Thus, using conditional probability, the

chance that he chose the unfair coin are

$$\frac{1}{\frac{1}{16}+1} = \boxed{\frac{16}{17}}.$$

24. On each edge of the $10 \times 10 \times 10$ cube, there are 10 cubes. The first and last cubes are corner cubes, and have 3 faces painted. The middle 8 have two faces painted. There are 8 such cubes per edge and 12 edges, thus 96 cubes in total. The only other cubes are the cubes part of the 8×8 center square on each face, which only have one face painted. Thus, the answer is $\boxed{96}$.

25. If we want to find the probability that event A happens given that event B happens, we have to divide the probability of both events happening by the probability of event B happening. (This is how we can calculate such "conditional" probabilities.) So, in this case, event A is choosing a student named Jim, while event B is choosing a student whose name begins with J.

    Hence, the chance of A and B both occurring is equal to $\frac{5}{18}$, since there are 5 students whose name is Jim and Jim starts with J. Also, the chance of B occurring is equal to $\frac{11}{18}$, since there are 11 students whose name begins with J. Therefore, our answer is equal to

$$\frac{\text{Chance of A and B}}{\text{Chance of B}} = \frac{\frac{5}{18}}{\frac{11}{18}} = \boxed{\frac{5}{11}}.$$

    Alternatively, note that there are 11 students whose names start with J, 5 of whom are named Jim. Therefore our probability is $\boxed{\frac{5}{11}}$.

26. If an integer ends in a zero, then it must be divisible by 10, and therefore also divisible by 5. Therefore, the smallest such number is simply $5 \times 24 = 120$. $120 = 2^3 \cdot 3^1 \cdot 5^1$. Therefore, the number of factors of 120 is $(3+1)(1+1)(1+1) = \boxed{16}$.

27. Because $\triangle ABX$ is isosceles, we have $\angle BAX = \angle ABX = 20°$. So, we have $\angle BXA = 180° - 20° - 20° = 140°$. In addition, $\triangle AXC$ is isosceles, and thus $\angle AXB = \angle ACX = 180° - 140° = 40°$. Thus finally, $\angle XAC = 180° - 40° - 40° = \boxed{100°}$.

28. The volume of a cylinder with radius r and height h is $\pi r^2 h$. Therefore, if both the radius and height are doubled, the volume will be $2^2 \times 2 = 8$ times greater. The new cake now feeds 64 people, or $\boxed{56}$ more than the original 8 people.

29. The sum of the columns and rows takes into account every entry twice, so no matter what arrangement the numbers are in, the sum of all the columns and all the rows is $2(1+2+\ldots+9) == 90$. Thus, the sum of the diagonals is $124 - 90 = \boxed{34}$.

30. If 1 gosling is born out of every 102 duck eggs, then $10404 \div 102 = 102$ goslings are born out of 10404 duck eggs. One-third of these is an ugly duckling, so there are $102 \div 3 = \boxed{34}$ ugly ducklings.

31. Note that $1 + \frac{1}{n} = \frac{n+1}{n}$. Thus, we get that

$$\left(1+\frac{1}{1}\right)\left(1+\frac{1}{2}\right)\cdots\left(1+\frac{1}{2013}\right) = \frac{2}{1}\cdot\frac{3}{2}\cdot\frac{4}{3}\cdots\frac{2014}{2013} = \frac{2014}{1} = \boxed{2014},$$

    since all of the numerators and denominators except for the first and last cancel.

32. If both numbers were odd, then their sum would be even. Therefore, one of the numbers must be even. The only even prime is 2, so the larger number is $2013 - 2 = \boxed{2011}$.

33. Let the side length be x. Then its surface area is $6x^2$ and it's side length is x, so using the fact that the surface area is 6 times its side length, then $6x^2 = 6x \implies x = 1$. The volume is x^3 which in this case is just $1^3 = \boxed{1}$.

34. The diagonal of this rectangle will be the diameter of the circle. Since the side lengths are 6 and 8, by the Pythagorean Theorem, the diameter will be $\sqrt{6^2 + 8^2} = 10$. The area of the circle is then $\pi r^2 = \pi \left(\frac{10}{2}\right)^2 = \boxed{25\pi}$.

35. Suppose the three numbers chosen are a, b, c with $a < b < c$. Then notice that even if a is the largest term less than b and that if c were the least term greater than b, then $a + b = c$. This fails the Triangle Inequality, which says that if a, b, c are the sides of a triangle, then $a + b > c$. So no such numbers work and the probability is $\boxed{0}$.

36. We can break this problem into cases depending on the number of elements in each subset. We will have 5 different cases since a subset can contain either 1 element, 3 elements, 5 elements, 7 elements, or 9 elements.
Case 1: Each subset contains only 1 element.

There are 10 ways to create a set with only 1 elements.
Case 2: Each subset contains 3 elements.

There are 10 ways to pick the first element, 9 ways to pick the second element and 8 ways to pick the third element. However since order does not matter, we have over counted. Thus we must divide $10 * 9 * 8$ by the number of ways to arrange 3 elements, which is $3 * 2 * 1$. So our answer is $\frac{10 * 9 * 8}{3 * 2 * 1}$ or 120.
Case 3: Each subset contains 5 elements.

There are 10 ways to pick the first element, 9 ways to pick the second element, 8 ways to pick the third element, 7 ways to pick the fourth, and 6 ways to pick the fifth element. However since order does not matter, we have over counted once again. Thus we must divide $10*9*8*7*6$ by the number of ways to arrange 5 elements, which is $5*4*3*2*1$. So our answer is $\frac{10 * 9 * 8 * 7 * 6}{5 * 4 * 3 * 2 * 1}$ or 252.
Case 4: Each subset contains 7 elements.

Note that we can compute this case similar to how we did the previous cases, but we can exploit symmetry to get the answer more quickly. Note that every time you pick a subset containing 3 elements, you leave behind a subset containing 7 elements. Thus the number of subsets of 7 elements is equal to the number of subsets of 3 elements, which is 120.
Case 5: Each subset contains 9 elements.

By the logic of Case 4, we see that this case has the same number of subsets as Case 1 so this case has 10 subsets. Summing up the sums of all of the 5 cases, we have $10+120+252+120+10$, which is $\boxed{512}$, our final total.

Remark : Did you notice that $512 = \frac{1024}{2} = \frac{2^{10}}{2}$, and that 10 was the number of elements in our original set? Do you think that this is a coincidence?

37. If we "unfold" the cube to form a flat net, we remember that the shortest distance from one corner to the opposite corner is a straight line. In this case, this line is the hypotenuse of a right triangle with legs 1 and 2, thus the minimum distance is $\boxed{\sqrt{5}}$.

38. Consider $7^2 = 49$. $7^4 = 49^2$ ends in 01. Note that the hundreds digit never matters. We have that 7^4 ends in 01, so since $7^{16} = (7^4)^4$, our desired answer is also $\boxed{01}$.

39. This is a classic stars and bars problem. We want to distribute 10 indistinguishable balls into 3 distinguishable boxes, each of which has at least one ball. This is because a, b, and c are positive, not just nonnegative. Therefore, put 1 ball into each box first. Now, we need to distribute 7

balls into 3 distinguishable boxes. In order to do this, we use 7 stars and 2 bars. Therefore, we have a total of $\binom{7+2}{2} = \binom{9}{2} = 36$ distributions. This means that there are 36 triplets (a, b, c) in positive integers to the equation $a + b + c = 10$.

40. Clearly $x = 0$ does not satisfy the equation. Thus, we can divide by x to get $x + \frac{1}{x} = 1$. Squaring this equation gives $x^2 + \frac{1}{x^2} + 2 = 1$, and multiplying both sides by x^2 yields $x^4 + x^2 + 1 = 0$. Therefore, $a = \boxed{1}$.

41. Note that the coordinates of the reflection of his house with respect to the river is $(5, -12)$. If we call this point H', then we see that from any point on the river, Dennis will be equidistant from both his house and the point H'. Therefore, the minimum distance he must travel from the origin, to the river, then to his house is equal to the distance from the origin, to the river, to H'.

    Lastly, we observe that the shortest path between two points is a straight line. So, this minimum distance is achieved if Dennis travels from the origin to H' in a straight line, meeting the river along the way. And by the Distance Formula, this distance is equal to $\sqrt{(5-0)^2 + (-12-0)^2} = \boxed{13}$.

42. Let the sum be S. Then $S = 3 + \frac{3}{S}$. Multiplying by S on both sides and rearranging, we obtain $S^2 - 3S - 3 = 0$. By the quadratic formula, the answer, which is clearly positive, must be $\boxed{\frac{3+\sqrt{21}}{2}}$.

43. Let $\overline{AB} = x$, and extend the lines BC, DE, FA. We see that we form an equilateral triangle with side length $4x$. Since the area of an equilateral triangle with side length s is equal to $\frac{s^2\sqrt{3}}{4}$, the area of the hexagon is then $\frac{(4x)^2\sqrt{3}}{4} - 3(\frac{x^2\sqrt{3}}{4}) = \frac{13x^2\sqrt{3}}{4} = 52\sqrt{3}$. Solving reveals that $x = 4$, and so the perimeter is $x + 2x + x + 2x + x + 2x = 9x = \boxed{36}$.

44. We recall that the number of divisors of n with prime factorization $p_1^{e_1} \cdots p_k^{e_k}$ is equal to $(e_1 + 1)(e_2 + 1) \cdots (e_k + 1)$. Thus, for an integer to have 22 positive divisors, it must be of the form $a^{10}b^1$ or a^{21}, with a, b prime. Clearly, the former choice will yield a smaller answer, and so we can choose $a = 2$ and $b = 3$ in order to minimize the product. Thus our answer is $2^{10} \cdot 3^1 = \boxed{3072}$.

45. Instead of the process the problem describes, consider the following process: each square you draw, count up the amount of unit squares it contains. Do that for every square and add up all the numbers. In both processes, you'll end up with the same amount. First let's look at the 6×6 squares. There's one of them so in total that takes up 36 unit squares. Now onto the 5×5 squares. Let's look at the top left unit square of each 5×5 square. It can only be within the upper left 4 unit squares, otherwise the big 5×5 square wouldn't fit. So that gives is $4*5*5 = 100$ more unit squares. Now we move onto 4×4. The top left unit square of the 4×4 will only fit within the upper left 9 unit squares, so this gives us $9 * 4 * 4 = 144$ unit squares. Continuing this process, we get that the 3×3 gives us $3 * 3 * 16 = 144$, the 2×2 gives $2 * 2 * 25 = 100$ and the 1×1 gives $1 * 1 * 36 = 36$. Adding it all up, we get $36 + 100 + 144 + 144 + 100 + 36 = \boxed{560}$.

46. Let there be a_n good subsets of the set $\{1, 2, \ldots, n\}$. If 1 is in the subset, then 2 cannot be in the subset, and there are a_{n-2} ways to choose the remaining numbers in the subset. If 1 is not in the subset, then there are a_{n-1} ways to choose the remaining numbers in the subset. Therefore, $a_n = a_{n-1} + a_{n-2}$. Furthermore, $a_1 = 2$ and $a_2 = 3$. Therefore $a_3 = 5, a_4 = 8, a_5 = 13, a_6 = 21, a_7 = 34, a_8 = 55, a_9 = 89, a_{10} = \boxed{144}$. Note that these numbers follow the Fibonacci sequence.

47. They can either split into three groups of 2, two groups of 3 or one group of 2 and one group of 4. In the first case, there are $\binom{6}{2}\binom{4}{2}\binom{2}{2} = 90$ ways. In the second case, there are $\binom{6}{3} = 20$ and in the final case there are $\binom{6}{2} = 15$ ways. Adding this up, we get a total of $90+20+15 = \boxed{125}$ ways.

48. This is a classic "stars and bars" problem. We want to order 3 bars and 10 stars. The stars to the left of the first bar will represent the balls that go into the first box. The stars to the right of the first bar but left of the second bar will represent the balls that go into the second box. The stars to the right of the second bar but left of the third bar will represent the balls that go into the third box. The stars to the right of the third bar will represent the balls that go into the fourth box. There are $\binom{10+3}{3} = \binom{13}{3} = \boxed{286}$ such orderings, which is the desired result.

49. Consider a coordinate plane where the x-axis goes from 0 to 60 and represents the amount of time in minutes after 1 that AJ arrives. Define the y-axis similarly for Mark. Each point in this coordinate plane represents two points in time: when AJ arrives and when Mark arrives. The region that represents the points in which AJ and Mark will be able to meet is bounded by the lines $x = 0$, $y = 0$, $x = y + 10$, $y = x + 10$, $x = 60$, and $y = 60$. For example, $x = y + 10$ is a line that bounds the region because x, which is the time that AJ arrives, can be at most 10 after the time that Mark arrives. The area of this region divided by the area of the entire square will be the desired answer. We can subtract the two unwanted regions, which are both isosceles right triangles with leg lengths of 50, from the entire square, in order to find the area of the desired region. This is $2 \times \frac{50 \times 50}{2} = 2500$. Therefore, the desired answer is $\frac{3600 - 2500}{3600} = \boxed{\frac{11}{36}}$.

50. Because $9 = x + y + z$ we can multiply the left side of the second equation by $\frac{x+y+z}{xyz}$ and the right hand side by $\frac{9}{xyz}$. The equation now becomes $16\frac{x+y+z}{z} + 9\frac{x+y+z}{y} + 4\frac{x+y+z}{x} = 81$. Rewriting this gives us that $(16\frac{x}{z} + 4\frac{z}{x}) + (9\frac{x}{y} + 4\frac{y}{x}) + (16\frac{y}{z} + 9\frac{z}{y}) = 52$. Now here comes the very tricky part. Note that no matter what real numbers a and b, we always have $(\sqrt{a} - \sqrt{b})^2 \geq 0$. Rearranging, this means that $a + b \geq 2\sqrt{ab}$. So now let's go back to our equation. In each pair of parentheses, let the expression left of the plus sign be a and the expression to the right be b. Now apply our cool result that $a + b \geq 2\sqrt{ab}$. For example, we could see that $16\frac{x}{z} + 4\frac{z}{x} \geq 2\sqrt{16 \cdot 4 \cdot \frac{z}{x}\frac{x}{z}} = 2 \cdot (8) = 16$. Repeating this for the second and third expression, we can see that $9\frac{x}{y} + 4\frac{y}{x} \geq 12$ and $16\frac{y}{z} + 9\frac{z}{y} \geq 24$. Thus, their sum is greater than or equal to $16 + 24 + 12 = 52$. But wait. We already knew that their sum was equal to 52, so how does this help? Well, in each of those inequalities we had, you'll remember that we got them all from using the fact that $(\sqrt{a} - \sqrt{b})^2 \geq 0$. But when is it exactly 0? When $a = b$. So since our sum is exactly 52, we need each of our "a"'s and "b"s to be equal. So we can solve the equations $16\frac{x}{z} = 4\frac{z}{x}$, $9\frac{x}{y} = 4\frac{y}{x}$ and $16\frac{y}{z} = 9\frac{z}{y}$. To solve the equations completely though, we must also use $x + y + z = 9$. This is a lot of tedious algebra, but in the end we should get that $x = 2, y = 3, z = 4$ which means that $xyz = \boxed{24}$.