

1. Recall that  $10^n = \underbrace{100 \dots 00}_{n \text{ zeros}}$ , hence  $10^{10} + 10^8 + 10^6 + 10^4 + 10^2 + 10^0 =$   
 $\boxed{10101010101}$ .

2. There are simply  $6 - 4 = \boxed{2}$  apples remaining.

3. Notice that  $9 \cdot 18 = 162$  and  $9 \cdot 19 = 171$ . Any other multiple of 9 will be further away from 169, so our answer is the closer of the two -  $\boxed{171}$ .

4. Since  $\frac{3+6+9}{2+4+6} = \frac{18}{12} = \frac{3}{2}$  and  $\frac{2+4+6}{3+6+9} = \frac{12}{18} = \frac{2}{3}$ , the answer is  $\frac{3}{2} - \frac{2}{3} = \boxed{\frac{5}{6}}$ .

5. Since every flower has 7 petals, and Alex has 1001 petals, there are  $\frac{1001}{7} =$   
 $\boxed{143}$  flowers.

6. There are 16 words in the sentence, and exactly 5 of them have four letters, as shown: “**What** is the probability **that** a randomly chosen **word** of **this** sentence has exactly **four** letters?”

Therefore, the desired probability is simply  $\boxed{\frac{5}{16}}$ .

7. 5 years ago, I was 8 years old, at which point my brother must have been 4 years old. Thus, today he is  $\boxed{9}$  years old.

8. Applying the definition, we have  $2\#0 = 2 \cdot 0 - 2 - 3 = -5$  and  $1\#4 = 1 \cdot 4 - 1 - 3 = 0$ . Thus,

$$(2\#0)\#(1\#4) = 5\#0 = (-5) \cdot 0 - (-5) - 3 = \boxed{2}.$$

9. Every time Leo says a word, there will either be 1 bark or 2 barks. Hence there are at least 15 barks. Every time Leo says “puppies” this number increases by 1, hence he said “puppies”  $22 - 15 = \boxed{7}$  times.

10. We can prime factorize  $12 = 2^2 \cdot 3$  and  $18 = 2 \cdot 3^2$ . Hence, since the greatest common divisor takes the smallest exponent of each prime, we have  $\gcd(12, 18) = 2 \cdot 3 = 6$ . Similarly, since the least common multiple takes the largest exponent of each prime, we have  $\text{lcm}(12, 18) = 2^2 \cdot 3^2 = 36$ . Thus our answer is  $6 + 36 = \boxed{42}$ .

Alternatively, we can utilize the Euclidean Algorithm to find  $\gcd(12, 18) = \gcd(12, 6) = \gcd(6, 6) = 6$ , then use  $\text{lcm}(x, y) = \frac{xy}{\gcd(x, y)}$  to find  $\text{lcm}(12, 18) = \frac{12 \cdot 18}{6} = 36$ . Our answer is then the same as before.

11. Evan’s 12th grade score was greater than  $8 \cdot 5 = 40$ , but was less than 42. The only integer between these is  $\boxed{41}$ .

12. After 5 days, the hedgehog has made  $5 \cdot 5 = 25$  new friends, for a total of  $4 + 25 = \boxed{29}$ . Alternatively, in general the hedgehog will have  $5(n-1) + 4 = 5n - 1$  friends on day  $n$  by the same logic, so when  $n = 6$  he will have 29 friends.
13. The 7 smallest prime numbers are 2, 3, 5, 7, 11, 13, and 17, the sum of which is  $\boxed{58}$ .
14. Let the number in question be  $x$ . Then we know

$$\begin{aligned}21 + \frac{1}{4}x &= \frac{3}{5}x \implies 21 = \left(\frac{3}{5} - \frac{1}{4}\right)x \\ \implies 21 &= \left(\frac{12}{20} - \frac{5}{20}\right)x = \left(\frac{7}{20}\right)x \\ \implies x &= 21 \cdot \frac{20}{7} = \boxed{60}.\end{aligned}$$

Verifying,  $\frac{1}{4}$  of 60 is 15, and  $\frac{3}{5}$  of 60 is 36. As  $21 + 15$  is indeed equal to 36, this check is successful.

15. We have  $0^2 = 0, 1^2 = 1, 2^2 = 4, 3^2 = 9, 4^2 = 16, 5^2 = 25, 6^2 = 36, 7^2 = 49, 8^2 = 64, \text{ and } 9^2 = 81$ . Any other perfect squares will have the same last digit as one of these, so the possible last digits are 0, 1, 4, 5, 6, and 9. There are  $\boxed{6}$  of these.
16. Since five students take both languages,  $27 - 5 = 22$  students take French only and  $32 - 5 = 27$  students take Spanish only. Hence a total of  $22 + 27 = \boxed{49}$  students take exactly 1 language course.
17. Suppose there are  $g$  girls in the room. Then there are  $2g$  teachers and  $g + 6$  boys in the room, for a total of  $4g + 6 = 38$  people. Thus  $g = 8$ , and there are  $8 + (8 + 6) = \boxed{22}$  children.
18. If Alex writes  $x$  more problems, then he will have written a total of  $61 + x$  of the  $187 + x$  problems. Hence

$$\begin{aligned}\frac{61 + x}{187 + x} &= \frac{1}{2} \\ \implies 122 + 2x &= 187 + x \\ \implies x &= \boxed{65}\end{aligned}$$

19. Since 2 darps is equal to 4 derps, 6 darps is equal to 12 derps. Similarly, since 3 derps is equal to 5 dirps, 12 derps is equal to 20 dirps. Hence, 6 darps is equivalent to  $\boxed{20}$  dirps.

20. Since the length of Alex's rectangle and the width of Alex's rectangle are both 3 times the length and width of Kelvin's rectangle, the area of Alex's rectangle is  $3 \cdot 3 = 9$  times the area of Kelvin's rectangle. Hence the area of Alex's rectangle is  $9 \cdot 12 = \boxed{108}$ .
21. We have  $2 \star 2 = 2^2 + 2^2 = 4 + 4 = 8$ , and thus  $2 \star (2 \star 2) = 2 \star 8 = 2^8 + 8^2 = 256 + 64 = \boxed{320}$ .
22. To mix the color purple, Rita would have to roll red once and blue once. She can either roll red first and blue next, which has a probability of  $\frac{3}{6} \cdot \frac{1}{6} = \frac{1}{12}$ , or roll blue first and red next, which also has a probability of  $\frac{1}{6} \cdot \frac{3}{6} = \frac{1}{12}$ . The final probability is thus  $\frac{1}{12} + \frac{1}{12} = \boxed{\frac{1}{6}}$ .

23. Let his final exam score be  $x$ . Then

$$\begin{aligned} \frac{91 + 89 + 88 + 94 + 87 + 85 + x}{7} &= x \\ \implies \frac{534 + x}{7} &= x \\ \implies 534 + x = 7x &\implies 534 = 6x \\ \implies x &= \boxed{89} \end{aligned}$$

24. Let the integers be  $x - 2, x - 1, x, x + 1, x + 2$ . Then  $5x = 210 \implies x = 42$ , so the largest number is  $x + 2 = 42 + 2 = \boxed{44}$ .
25. Suppose the side length of the cube is  $s$ . Then the area of each face of the cube, being a square, has area  $s^2$ . Since there are 6 such faces, the surface area of the cube is  $6s^2$ . As such we have  $6s^2 = 294 \implies s^2 = 49 \implies s = \pm 7$ . Clearly  $s = -7$  does not make sense as a side length, leaving the solution  $s = \boxed{7}$ .
26. Suppose there are  $a$  big peaches and  $b$  little peaches. Hence there are  $a + b$  peaches in the pile. Considering their weights,  $8a + 4b = 252 \implies 2a + b = 63$ . Thus  $a + b = 63 - a$ , so minimizing  $a + b$  is the same as maximizing  $a$ . The largest possible value of  $a$  is 31, so there are  $63 - 31 = \boxed{32}$  peaches in the pile.
27. In the 5 minutes that James has a hammer, he can crush 150 candies. After that, it takes him 4 seconds to crush one candy, so crushing another 30 candies takes 120 seconds - or two minutes. Thus he needs at least  $5 + 2 = \boxed{7}$  minutes to crush 180 candies.
28. Let  $a$  and  $b$  be the integers, and without loss of generality assume that  $a \geq b$  (if  $a < b$ , we can simply switch the two). Since  $8 = a + b \leq 2a$ , we know that  $a$  is at least 4. Furthermore, since  $a^2 \leq a^2 + b^2 = 34$ , we know that  $a$  is less than 6. Therefore we only need to check  $a = 4$  and

- $a = 5$ , the latter of which leads to the solution  $a = 5, b = 3$ . Therefore,  $ab = 5 \cdot 3 = \boxed{15}$ .
29. The total score of the class beforehand is  $12 \cdot 65 = 780$ , and the total score afterwards is  $13 \cdot 66 = 858$ . Hence the new student scored a  $858 - 780 = \boxed{78}$  on the test.
30. Suppose there are  $a$  alpacas and  $c$  chickens. Then  $a + 2c = 94$  and  $2c + 4a = 238$ , so  $3a = 238 - 94 = 144$ . Hence  $a = 48 \implies b = 23$ , and thus there are  $\boxed{71}$  animals on the farm.
31. Nikita catches up to Lev at the rate of 4 miles per hour, so it takes her  $\boxed{15}$  minutes before she is half a mile ahead of Lev.
32. Though the top floor is labeled "88", we have skipped all the floors containing a 4. That includes the floors 4, 14,  $\dots$ , 84 and 40, 41,  $\dots$ , 49, of which there are 18 (there are 9 in the first list and 10 in the second, but both lists contain 44). Hence there are actually only  $88 - 18 = \boxed{70}$  floors in this building.
33. The squares less than 20 are  $0^2 = 0, 1^2 = 1, 2^2 = 4, 3^2 = 9$ , and  $4^2 = 16$ . There are  $\binom{5}{2} + 5 = 15$  combinations of these, but  $16 + 16$  and  $9 + 16 > 20$  and  $0 + 0 < 1$ . Hence  $15 - 3 = \boxed{12}$  of these lie between 1 and 20. Notice that it is also easily verifiable that no combinations result in the same sum.
34. 5 sea otters can eat only half the amount that 10 sea otters can, but since they have  $\frac{3}{2}$  the time the 10 sea otters had, they can eat  $\frac{1}{2} \cdot \frac{3}{2} = \frac{3}{4}$  the amount. Hence they eat  $36 \cdot \frac{3}{4} = \boxed{27}$  sea urchins.
35. The remainder upon dividing 9 by 2 is 1, so  $9\%2 = 1$ . Similarly, the remainder upon dividing 9 by 4 is 1, so  $9\%4 = 1$ . Hence  $9\Delta 2 = \frac{1+1}{2} = \boxed{1}$ .
36. 7 people is not enough to guarantee this, as they could each be born on a different day of the week. However, adding one more person to that mix ensures that some two people will be born on the same day of the week, so  $\boxed{8}$  people is the least number necessary.
37. The biking leg of his trip is completed in  $\frac{2}{10} = \frac{1}{5}$  of an hour, or 12 minutes, while the swimming leg of his trip is completed in  $\frac{3}{12} = \frac{1}{4}$  of an hour, or 15 minutes. Since the entire trip takes  $\frac{6}{6} = 1$  hour, he should spend  $60 - 12 - 15 = \boxed{33}$  minutes on his walk.
38. Since the triangle's altitudes are all equal, the triangle must be equilateral. Suppose the side length of this triangle is  $2s$ . Then, by the Pythagorean theorem,  $s^2 + 6^2 = (2s)^2 \implies 36 = 3s^2 \implies 12 = s^2$ . The area of the triangle is thus  $\frac{s^2\sqrt{3}}{4} = \frac{12\sqrt{3}}{4} = \boxed{3\sqrt{3}}$ .

39. Let the squares be  $(x-2)^2$ ,  $(x-1)^2$ ,  $x^2$ ,  $(x+1)^2$ , and  $(x+2)^2$ . Their sum is  $5x^2 + 2^2 + 1^2 + 1^2 + 2^2 = 5x^2 + 10$ , hence  $5x^2 + 10 = 146 \cdot 5 = 730$ . Thus  $5x^2 = 720 \implies x^2 = 144 \implies x = \pm 12$ , making the smallest of these squares  $\boxed{100}$ .
40. If  $x \leq 4$ , then the median of the numbers is 4 and hence  $\frac{24+x}{5} = 4 \implies x = -4$ . If  $4 < x < 7$ , then the median of the numbers is  $x$  and hence  $\frac{24+x}{5} = x \implies x = 6$ . If  $x \geq 7$ , then the median of the numbers is 7 and hence  $\frac{24+x}{5} = 7 \implies x = 11$ . The sum of these three values is thus  $(-4) + 6 + 11 = \boxed{13}$ .
41. Since the triangle is isosceles, some two of its side lengths are equal. Hence either  $x - 4 = 2x - 9 \implies x = 5$ ,  $2x - 9 = 3x - 15 \implies x = 6$ , or  $x - 4 = 3x - 15 \implies x = \frac{11}{2}$ . However, if  $x = 5$ , the side lengths of the triangle are 1, 1, and 0 - clearly not a triangle. Hence the only two valid values of  $x$  are 6 and  $\frac{11}{2}$ , the sum of which is  $\boxed{\frac{23}{2}}$ .
42. Since  $2014 = 2 \cdot 19 \cdot 53$ , the sum of its factors is  $(1+2)(1+19)(1+53) = 3 \cdot 20 \cdot 54 = \boxed{3240}$ .
43. The only way that this is possible is if the children alternate in the form "GBGBGBG", where "G" denotes a girl and "B" denotes a boy. The 4 girls can arrange themselves in any of  $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$  ways, while the 3 boys can arrange themselves in  $3! = 3 \cdot 2 \cdot 1 = 6$  ways. Thus there are a total of  $6 \cdot 24 = \boxed{144}$  possible arrangements.
44. We will first count the numbers from 1 to 1999 that do *not* have a 6 in it. There are  $2 \cdot 9 \cdot 9 \cdot 9 - 1 = 1457$  such numbers from 1 to 1999, as there are two choices (0 or 1) for the thousands digit and nine for each of the hundreds, tens, and ones digit (excluding the case of 0000), so there are  $1999 - 1457 = 542$  numbers between 1 and 1999 that do have a 6 in it. Adding in the final case of 2006, there are  $\boxed{543}$  such numbers between 1 and 2014.
45. Let the number be  $\overline{abc}$ . If any of  $a, b, c$  are zero, then the product of the digits would be zero, implying that the sum of the digits is 0 - impossible. Hence  $a, b, c$  are all positive digits. WLOG assume that  $a \leq b \leq c$  - we'll account for the permutations later. Thus we have  $abc = a + b + c \leq 3c \implies ab \leq 3$ , meaning we need check only  $(a, b) = (1, 1)$ ,  $(a, b) = (1, 2)$ , and  $(a, b) = (1, 3)$ . The former case gives us  $c = 2 + c$ , contradiction, the second gives us  $2c = 3 + c \implies c = 3$ , and the third gives us  $3c = 4 + c \implies c = 2$ , a contradiction as we assumed  $b \leq c$ . Therefore the only possible numbers are the permutations of  $\overline{123}$ , of which there are  $3! = \boxed{6}$ .
46. There are 105 multiples of 19 between 1 and 2014, 53 of which are odd and 52 of which are even. Hence there is  $\boxed{1}$  more number that is yellow and blue than yellow and orange.

47. By symmetry, the probability of hearing a “bong”  $n$  times is equal to the probability of hearing a bong  $2015 - n$  times. Hence the probability of hearing a “bong” anywhere from 1008 to 2015 times, inclusive, is equal to the probability of hearing a “bong” anywhere from 0 to 1007 times, inclusive. Since these account for all the possibilities, the probability of each of these is  $\boxed{\frac{1}{2}}$ .
48. Because  $18 > 12 \implies 3\sqrt{2} > 2\sqrt{3} \implies \frac{\sqrt{2}}{2} > \frac{\sqrt{3}}{3}$ , we have  $A > B$ . Since  $98 < 100 \implies 7\sqrt{2} < 10 \implies \frac{\sqrt{2}}{2} < \frac{5}{7}$ , we have  $A < C$ . Finally, since  $12 > 9 \implies 2\sqrt{3} > 3 \implies \frac{\sqrt{3}}{3} > 0.5$ , hence  $D < B$ . Thus  $D < B < A < C$ , and our answer is  $\boxed{DBAC}$ .
49. Plugging in  $x = 1$  and  $x = 2$  gives us  $f(1) + 1 \cdot f(2) = 2$  and  $f(2) + 2 \cdot f(1) = 2$ , hence  $f(1) + f(2) = f(2) + 2f(1) \implies f(1) = \boxed{0}$ .
50. There are  $52 \cdot 51$  possible positions for these two cards. If they are next to each other, there are 51 possible places they could be in, and there are 2 ways to arrange them within each place. Hence the desired probability is  $\frac{2 \cdot 51}{51 \cdot 52} = \frac{2}{52} = \boxed{\frac{1}{26}}$ .