Joe Holbrook Memorial Math Competition

4th Grade

October 9th, 2016

General Rules

- You will have **90 minutes** to solve **50** questions. Your score is the number of correct answers.

- Only answers recorded on the answer sheet will be graded.

- This is an individual test. Anyone caught communicating with another student will be removed from the exam.

- Scores will be posted on the website. Please do not forget your ID number, as that will be the sole means of identification for the scores.

- You may **not** use the following aids:

    - Calculator or other computing device
    - Compass
    - Protractor
    - Ruler or straightedge

    In addition, you must use the scrap paper supplied by the proctors.

Other Notes

- Write legibly. If the graders cannot read your answer, you will be given no credit for that question.

- Fractions should be written in **lowest terms**. Please convert all mixed numbers into **improper fractions**.

- For constants such as e or π, do not approximate your answer: for example, if the answer to a question is 7π, then you should **not** write 22 or 21.99.

- You do not need to write units in your answers.

- Rationalize all denominators. In addition, numbers within a square root must be squarefree, e.g. $\sqrt{63}$ should be written as $3\sqrt{7}$.

- Ties will be broken by the number of correct responses to questions 41 through 50. Further ties will be broken by the number of correct responses in the last five questions.

1. Kelvin the Frog's favorite song is 6 minutes long. How many times can he listen to the song in half an hour?

2. Compute $-17 \times -16 \times -15 \times \cdots \times 15 \times 16 \times 17$?

3. Alex the Kat has written 25 math problems. If he wants to write 40 in total. how many does he have left to write?

4. The Bergen Tech soccer team won a match 5 to 2. How many goals were scored in all?

5. How many more sides does a hexagon have than a triangle?

6. Zack made half of his eight 3-point shots. How many points did he score?

7. What fraction of months start with a J?

8. If Mr. Pinyan grades 150 tests at the end of each of the three trimesters, how many tests does he grade in a year?

9. If the Bergen County Academies' field has an area of 5100 square yards and one third of the field is covered in geese droppings, what is the total area of the clean regions of the field?

10. If Yousun is 5 feet tall and Youjung is 6 inches taller than Yousun, how tall is Youjung in inches?

11. Compute the sum $123 + 12.3 + 1.23$.

12. A phone has a maximum battery life of 10 hours. There is 12% battery left. How many minutes are left until the phone dies?

13. What is the value obtained when 4^4 is divided by 2^2?

14. What is the value of $\dfrac{1}{2} \times \dfrac{2}{3} \times \dfrac{3}{6}$?

15. Compute $\dfrac{5}{55} \times 5555$.

16. When Kelvin the Frog was a tadpole, he took the JHMMC. His score improved by the same number of questions every year. If he got 31 questions right in 4th grade and 43 questions right in 8th grade, how many did he get right in 7th grade?

17. Harry Hounini, the famous magician, invents a magic trick, and he hopes you will test it out for him. This is how the trick works: first think of any number. Now add that number to 2016, and multiply the sum by 4. Now subtract 12 from the product, and divide the result by 4. Lastly, subtract off the original number. Incredibly, Hounini knows exactly what that number is. What is it?

18. What is the value of $\dfrac{1}{2} \times \dfrac{8}{16} \times \dfrac{17}{34} \times \dfrac{50}{100}$?

19. The number of lilypads in Kelvin the Frog's pond doubles every day. If there were 48 lilypads on Saturday, on what day of the week did he have an odd number of lilypads?

20. In a ping-pong game that ends when a person hits 21 points, Marvin and June scored 34 points together. If Marvin won, how many points did June score?

21. Compute $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + ... + 63$.

22. A fruit smoothie has 10% water. A soda has 2% water. If 1 liter of the smoothie and 1 liter of the soda are mixed, what percent of the resulting solution is water?

23. Jake bought a bad batch of eggs, and some of his cakes have gone bad. There was a total of 132 cakes, but two out of every three were bad. If he threw out all of the bad cakes, how many does he have left?

24. Evaluate $1 + 3 + 5 + \ldots + 17 + 19 + 31 + 33 + \ldots + 59$.

25. January 1st, 2016 was a Friday. What day of the week is the 2016th day after January 1st, 2016?

26. David and June went to a pizza restaurant together. David ordered 3 slices of plain pizza and 2 slices of pepperoni pizza. June ordered 2 slices of plain pizza and 4 slices of pepperoni pizza. If David and June paid $25 in total and a slice of pepperoni pizza costs 50 cents more than a slice of plain pizza, how much did David pay?

27. David Song is writing songs. If one of his songs is 3 minutes and 45 seconds long, what fraction of an hour is it?

28. The Bergen County Academies Admissions Exam had 40 questions. Of three students that were accepted, their scores were 34, 35, and 39. Of three students that were not accepted, their scores were 23, 14, and 17. What is the difference between the average scores of the accepted students and the students who were not accepted?

29. Suppose 3 flips are worth 5 flops and 9 flops are worth 14 flaps. How many flaps are equal to 54 flips?

30. What is the LCM (least common multiple) of 8, 78 and 130?

31. Letter blocks with the letters "A", "B", and "C" are in a bag. If you take out a letter at a time, what is the probability that they come out in the order spelling "BCA"?

32. Andrew, Ben, Caleb, Dennis, and Eddie are working to build a house. Sadly, Andrew and Dennis are lazy, so they only work for 1 hour. They are also mean, so the other three people don't work with them. While Andrew and Dennis work alone, they finish only 3% of the house. Caleb, Dennis, and Eddie working together work 5 times as fast. How many minutes will it take them to finish building the house?

33. Arthur and Sunny are running a 100 meter race against each other. For the first 5 seconds, Arthur runs at 8 m/s. Tired, he slows down to 3 m/s for the rest of the race. Sunny opened the first 8 seconds of the race running at 4 m/s. If he wants to at least tie Arthur, what is the minimum speed he must run at for the rest of the race?

34. Ryan and Max love playing Candy Crush. Ryan clears 5 levels everyday. Max cleared 1 level on the first day and clears one more level each day. How many days will it take Max to catch up to Ryan?

35. How many distinct words (strings of letters) can be formed by using all of the letters $JHMMC$?

36. If $a\#b = \dfrac{a-2}{b}$, then what is the smallest positive integer value of n such that $(n\#3)\#3$ has a positive integer value?

37. Two similar right triangles have areas of 40cm^2 and 360cm^2 respectively. If the smaller has a hypotenuse of length 15cm, what is the length, in centimeters, of the hypotenuse of the larger triangle?

38. What is the largest integer value for x such that $\dfrac{x}{x+2} < \dfrac{61}{64}$?

39. Jenn and Julia live on the same street. One day, they both decide to go to the other's house. After leaving at the same time, they meet 200 meters from Jenn's house. Being shy, both of them continue walking without stopping. After arriving at their destinations, the two feel bad and walk back, hoping to meet each other and apologize. They eventually meet again, 100 meters from Julia's house this time. What is the distance between their houses?

40. A 150 liter water tank is empty. If a pipe fills the tank at a rate of 4 liters per minute, and water drips from a crack at a rate of 2.5 liters per minute. How many seconds will it take to fill the entire tank?

41. We are given a list of seven numbers a, b, c, d, e, f, g, such that the average of the first four numbers is 7 and the average of the last four numbers is 4. Given that the average of all 7 numbers in the set is 5, what is the value of d?

42. It takes a team of 10 workers 10 days to build a wall. 2 days into a project building 5 walls, the boss realizes they will not finish in time. How many workers must he add so that he can finish before the 10-day deadline?

43. How many ways are there to pick 3 non-negative integers such that the sum of the first two is 10 minus the third?

44. If an isosceles triangle has sides of 32 feet and 65 feet, what is the area of the triangle in square feet?

45. AJ the Dennis the Menace wants to start a soccer league where each team plays a game against every other team twice. If there are 90 games played in total, how many teams are playing in the league?

46. Given a positive integer x, what is the number of possibilities for the greatest common divisor of $5x+18$ and $8x+30$?

47. In a standard 8 by 8 chessboard, how many rectangles are there?

48. The degree measures of the angles in a convex, 12-sided polygon form an arithmetic sequence with integer values. Find the smallest possible value degree measure of the smallest angle.

49. Suppose $\dfrac{1}{1^2} + \dfrac{1}{2^2} + \dfrac{1}{3^2} + \cdots + \dfrac{1}{n^2} + \cdots = S$. In terms of S, what is $\dfrac{1}{1^2} + \dfrac{1}{3^2} + \dfrac{1}{5^2} + \cdots + \dfrac{1}{(2n+1)^2} + \cdots$?

50. Hannah and David Song each have two drips, the currency in DrizzleLand. Hannah tells David that if it rains the next day, she will give him one drip, and if the weather is sunny, David will give her one drip. David cleverly agrees to this deal because on any given day in DrizzleLand, it rains with probability 3/4 and is sunny with probability 1/4. They continue this process for days, until one of them becomes broke. What is the probability that David will run out of money?