Joe Holbrook Memorial Math Competition

7th Grade

October 9th, 2016

General Rules

- You will have **90 minutes** to solve **50** questions. Your score is the number of correct answers.
- Only answers recorded on the answer sheet will be graded.
- This is an individual test. Anyone caught communicating with another student will be removed from the exam.
- Scores will be posted on the website. Please do not forget your ID number, as that will be the sole means of identification for the scores.
- You may **not** use the following aids:
  - Calculator or other computing device
  - Compass
  - Protractor
  - Ruler or straightedge

  In addition, you must use the scrap paper supplied by the proctors.

Other Notes

- Write legibly. If the graders cannot read your answer, you will be given no credit for that question.
- Fractions should be written in **lowest terms**. Please convert all mixed numbers into **improper fractions**.
- For constants such as e or π, do not approximate your answer: for example, if the answer to a question is 7π, then you should **not** write 22 or 21.99.
- You do not need to write units in your answers.
- Rationalize all denominators. In addition, numbers within a square root must be squarefree, e.g. $\sqrt{63}$ should be written as $3\sqrt{7}$.
- Ties will be broken by the number of correct responses to questions 41 through 50. Further ties will be broken by the number of correct responses in the last five questions.

1. Kelvin the Frog's favorite song is 6 minutes long. How many times can he listen to the song in half an hour?
2. Compute $2 + (0 - (1 \cdot 6 \cdot (2^{0 \cdot (\frac{1}{6})})))$.
3. Alex the Kat has written 25 questions for the JHMMC. If he wants to write 40 in total. how many does he have left to write?
4. Zack made half of his eight 3-point shots. How many points did he score?
5. What is the value obtained when 4^4 is divided by 2^2?
6. Two regular polygons have angles of 135 degrees and 108 degrees. Find the sum of the number of sides on both polygons.
7. Find the greatest common factor of 2016 and 2772.
8. What is the smallest integer n such that $2^n > 2016$?
9. When Kelvin the Frog was a tadpole, he took the JHMMC. His score improved by the same number of questions every year. If he got 31 questions right in 4th grade and 43 questions right in 8th grade, how many did he get right in 7th grade?
10. The number of lilypads in Kelvin the Frog's pond doubles every day. If there were 48 lilypads on Saturday, on what day of the week did he have an odd number of lilypads?
11. In a ping-pong game that ends when a person hits 21 points, Marvin and June scored 34 points together. If Marvin won, how many points did June score?
12. January 1st, 2016 was a Friday. What day of the week is the 2016th day after January 1st, 2016?
13. If $f(x) = x + 1$, $g(x) = \lfloor x \rfloor$, and $h(x) = g(f(x))$, compute $h(\pi^2)$.
14. Jake goes to the grocery store because he needs to buy milk, eggs, and butter for a cake recipe. If there are 3 different brands of milk, 2 different brands of eggs, and 4 different brands of butter, and Jake only wants to buy one of each item, how many different combinations of milk eggs and butter can he buy?
15. What is the sum of the roots of $x^2 - 2x + 3$?
16. Suppose 3 flips are worth 5 flops and 9 flops are worth 14 flaps. How many flaps are equal to 54 flips?
17. Letter blocks with the letters "A", "B", and "C" are in a bag. If you take out a letter at a time, what is the probability that they come out in the order spelling "BCA"?
18. Arthur and Sunny are running a 100 meter race against each other. For the first 5 seconds, Arthur runs at 8 m/s. Tired, he slows down to 3 m/s for the rest of the race. Meanwhile, Sunny opened the first 8 seconds of the race running only at 4 m/s. If he wants to at least tie Arthur, what is the minimum speed he must run at for the rest of the race? Express answer as a common fraction.
19. In how many distinct words (strings of letters) can be formed by permuting all of the letters of $JHMMC$?
20. Compute the units digit of $2^{2016} + 3^{2016}$.
21. The area of a square is 25. The area of an equilateral triangle is $9\sqrt{3}$. What is the difference in side lengths?
22. What is the sum of the largest five digit number and smallest five digit number Tom can form, given that adjacent digits may not be both odd or both even, and adjacent digits must be at least two apart?
23. Andrew, Ben, and Caleb want to split their n coins. They agreed that Andrew should get $\frac{1}{2}$ of the coins, Ben should get $\frac{1}{3}$, and Caleb should get $\frac{1}{9}$, with the rest going to charity. However, their coins won't split evenly! Their friend Dennis comes and loans them a coin. Now, they split the coins as decided before, and one coin is left. Find n.
24. Two similar right triangles have areas of $40cm^2$ and $360cm^2$ respectively. If the smaller has a hypotenuse of length $15cm$, what is the length, in centimeters, of the hypotenuse of the larger triangle?
25. What is the largest integer value for x such that $\frac{x}{x+2} < \frac{61}{64}$?

26. Jake can make 12 mini cheesecakes in half an hour. If he works with his brother, Zach, they can make 84 cheesecakes in two hours. How many can Zach make in 4 hours?

27. Jake is once again making mini cheesecakes. They are cylinders with radius $\frac{3}{2}$ and height 1. If he feels generous one day and wants to increase the volume by 44% and keep the same height, what should he make the radius of the cakes?

28. How many numbers less than 2016 have exactly 3 factors?

29. Compute $|x+y|$ given that $(x+y)^4 - 4 \cdot (x+y)^2 = 1$ and $(x+y)^4 + 4 \cdot (x+y)^2 = 3$.

30. Everyone in DrizzleLand has a 1 in 150 chance of having the muggy virus. The test for this virus has a 96% of giving the correct result for any patient, whether or not he or she has the virus. If Zach tests positive, what is the probability that he actually has the muggy virus?

31. Arthur the Aardvark has a tight leash tethered to a stump located at the vertex of a regular pentagonal fence. Given that the fence has side length 4 meters and the leash is 3 meters long, what is the area of the region along which Arthur can travel?

32. The digits $4, 5, 6$, and 7 are permuted to create a four-digit number. What is the probability that this number is divisible by 4?

33. Given a circle with a radius of 6, what is the area of the largest square that can be inscribed in the circle?

34. Triangles ABC and XYZ are similar. It is known that $AB = 15$, $BC = 10$, $XY = 3$, and $\angle ABC = 30°$. Find the area of triangle XYZ.

35. Consider an 8 digit number using each of the digits $1 - 8$ exactly once, with the following conditions:

    The greatest common factor of the first two digits is greater than 1. The next two digits are both prime numbers. The next two digits are both perfect squares. The last two digits are both triangular numbers.

    What is the smallest number that satisfies the given conditions?

36. What is $1^2 - 3^2 + 5^2 - 7^2 + \cdots + 65^2 - 67^2$?

37. Right triangle ABC has a right angle at B and $\angle C$ has a measure of 30 degrees. D is on BC with $\angle ADB = 45$ degrees. If $CD = 2 - \sqrt{3}$, find BD.

38. Find the sum of all the positive even divisors of 2016.

39. Find the remainder when $1 + 2 + 4 + 8 + \cdots + 2^{2015}$ is divided by 7.

40. Compute the sum $\frac{7}{\sqrt{1}+\sqrt{2}} + \frac{7}{\sqrt{2}+\sqrt{3}} + \frac{7}{\sqrt{3}+\sqrt{4}} + \cdots + \frac{7}{\sqrt{48}+\sqrt{49}}$.

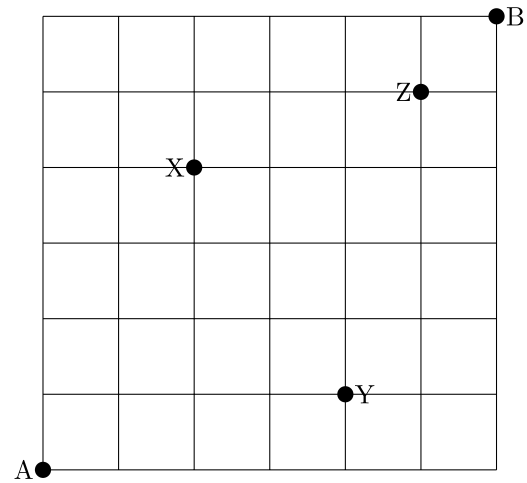
41. Arthur the Aardvark stands before 5 ant nests. Arthur is a highly trained sharpshooter, and thus, the probability that he hits a nest with a single shot of his tongue is $\frac{4}{5}$. He shoots at each of the nests. What is the probability that an even number of nests are hit?

42. Point O is selected inside a rectangle $ABCD$ such that $OA = 4$ and $OC = 11$. Find $OB^2 + OD^2$.

43. Let a, b, c, d be distinct digits. A real number is called "cool" if it can be represented as a repeating decimal of the form $0.abcd$. Find the sum of all "cool" numbers.

44. Determine the number of six-digit integers $\overline{810abc}$ such that each of the numbers $\overline{810abc}, \overline{810cab}, \overline{810bca}$ is divisible by 27.

45. Suppose we have the following grid. How many ways are there to get from A to B such that you don't pass through the points X,Y, and Z and such that you can only move right or up.



46. Let x be the length of the shortest path between the points $(4, 9)$ and $(12, 4)$ that touches both the x-axis and the y-axis at least once. Find x^2.

47. Let all k positive divisors of 2016^2 be $d_1, d_2, ...d_k$. What is the sum $\frac{1}{d_1 + 2016} + \frac{1}{d_2 + 2016} + ... + \frac{1}{d_k + 2016}$?

48. Alex the Kat is playing with his magical coloured yarn balls. They come in three colours: red, yellow, and blue. Every hour, the balls multiply and change colour with the following probabilities:

   - A red ball has probability $\frac{5}{12}$ of becoming two red balls, $\frac{1}{3}$ of becoming one red ball and one yellow ball, and $\frac{1}{4}$ of becoming one blue ball.
   - A yellow ball has probability $\frac{1}{2}$ of becoming two yellow balls, $\frac{1}{4}$ of becoming one yellow ball, and $\frac{1}{4}$ of becoming one blue ball;
   - A blue ball does not change colour.

   If Alex starts with one red yarn ball, what is the probability that eventually all balls will be blue?

49. In orthodiagonal quadrilateral $ABCD$, $AC = 12$ and $BD = 18$. Points M and N are the midpoints of AB and CD, respectively. If the projection from point M onto CD intersects CD at P and $NP = 6$, find MP.

50. In triangle $\triangle ABC$, the angle bisectors of angles A, B, C concur at the incenter I. Given that $AB = 13, BC = 14, CA = 15$, find the square of the area of a triangle with side-lengths AI, BI, CI.