Joe Holbrook Memorial Math Competition

4th Grade Solutions

October 9th, 2016

1. Since half an hour is 30 minutes, Kelvin the Frog can listen to his favorite song $\frac{30}{6} = \boxed{5}$ times.
2. The list of numbers consists of integers between -17 and 17, which includes 0. Any product of a number and 0 is $\boxed{0}$.
3. Alex the Kat needs to write x questions such that $25 + x = 40$, so $x = 40 - 25 = 15$.
4. A total of 5+2=$\boxed{7}$ points were scored.
5. A hexagon has 6 sides, and a triangle has 3 sides, so a hexagon has $6 - 3 = \boxed{3}$ more sides than a triangle does.
6. Zack made half of eight shots, meaning he made $0.5 \cdot 8 = 4$shots. Each shot was worth 3 points, so he made a total of $4 \cdot 3 = \boxed{12}$ points.
7. Three of the twelve months start with a J: January, June, July. Therefore, $\frac{3}{12} = \boxed{\frac{1}{4}}$.
8. The total tests graded can be calculated by mutiplying 3 and 150, which yields $\boxed{450}$.
9. If one third of the field is covered in geese droppings, that means $\frac{1}{3} \cdot 5100 = 1700$ square yards are covered in geese poop, leaving $5100 - 1700 = \boxed{3400}$ square yards of clean field.
10. There are twelve inches per foot, so Yousun is $5 \cdot 12 = 60$ inches tall. Youjung is therefore $60 + 6 = \boxed{66}$ inches tall.
11. Align the addends vertically by the decimal point and add the digits in each column to get $123+12.3+1.23 = \boxed{136.53}$.
12. The phone has a maximum battery life of $10 + 60 = 600$ minutes. Therefore, the phone has $12\% \cdot 600 = \boxed{72}$ minutes left.
13. We have $4^4 = 256$, and $2^2 = 4$, so the answer is $\frac{256}{4} = \boxed{64}$. Alternatively, note that $4 = 2^2$, so $\frac{4^4}{2^2} = \frac{4^4}{4} = 4^3 = \boxed{64}$.
14. Aligning our multiplicands in vertical fashion, we see that many numerators and denominators cancel, leaving a final answer of $\boxed{\frac{1}{6}}$.
15. $\frac{5}{55} = \frac{1}{11}$. $111 \cdot 5555 = \frac{5555}{11} = \boxed{505}$.
16. Four years pass between Kelvin's 4th and 8th grades, which means that his scores improved 4 times. During this period, his score increased by $43 - 31 = 12$ points. Since he increased by an equal amount every year, we divide the total increase by the total time to get $\frac{12}{4} = 3$. This means that in 7th grade he scored 3 fewer points than in 8th grade, or $43 - 40 = \boxed{40}$.
17. Let's call our number n. We then perform many operations: first we obtain $n + 2016$, then $4 \cdot (n + 2016)$, then $4 \cdot (n + 2016) - 12$, then $\frac{4 \cdot (n + 2016) - 12}{4} = n + 2016 - 3 = n + 2013$, then $(n + 2013) - n = \boxed{2013}$.
18. Each of the fractions here are equal to $\frac{1}{2}$. $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \boxed{\frac{1}{16}}$.

19. Since doubling any number gives us an even number, we have to work backwards. On Friday, Kelvin had $\frac{48}{2} = 24$ lilypads. On Thursday, he had $\frac{24}{2} = 12$ lilypads. On Wednesday, he had $\frac{12}{2} = 6$ lilypads. On Tuesday, he had $\frac{6}{2} = 3$ lilypads, making our day $\boxed{\text{Tuesday}}$.

20. Let the number of points Marvin scored be M and the number of points June scored be J. We are given that $M + J = 34$, and since Marvin won, $M = 21$. Therefore, $J = 34 - M = 34 - 21 = 13$. June scored $\boxed{13}$ points.

21. The sum of integers from 1 through n can be found by the formula $\frac{n(n+1)}{2}$. Plugging in $n = 63$ results in $\boxed{2016}$. For more information on this fascinating topic, look up "triangle numbers" – you'll encounter them often in the future!

22. One liter is also 1000 milliliters. One liter of the smoothe has $10\% \cdot 1000 = 100$ milliliters of water, and one liter of the soda has $2\% \cdot 1000 = 20$ milliliters of water. Therefore, when mixed, we have $100 + 20 = 120$ milliliters of water, and a total of $1000 + 1000 = 2000$ milliliters of liquid. So, $\frac{120}{2000} = \frac{6}{100} = \boxed{6\%}$.

23. Two out of every three cakes were thrown out, which means that one in every three cakes was kept. This is $\frac{1}{3}$ of 132, which is $\boxed{44}$.

24. This looks a lot like $1 + 3 + 5 + \ldots + 59$. Using the formula for the sum of the first n odds, n^2, we obtain $1 + 3 + 5 + \ldots + 59 = 30^2 = 900$. However, we don't have the terms 21 to 29. To remedy this error, we subtract $1 + 3 + 5 + \ldots + 29 = 15^2 = 225$. However, we subtracted the terms $1 + 3 + 5 + \ldots + 19$, which we shouldn't have, so add $1 + 3 + 5 + \ldots + 19 = 10^2 = 100$ back. $900 - 225 + 100 = \boxed{775}$.

25. Every 7 days after a Friday is also a Friday. Therefore, if you divide 2016 by 7, the quotient is irrelevant, and the remainder tells you how many days of the week you must count before reaching the correct day of the week. However, since 2016 is perfectly divisible by 7, the remainder is 0, so the day of the week will be the same. Therefore, the 2016th day after January 1st, 2016 will be on a $\boxed{\text{Friday}}$.

26. Let a slice of plain pizza cost x dollars and a slice of pepperoni pizza cost $x + 0.5$ dollars. David and June ordered 3+2= 5 slices of plain pizza and $2 + 4 = 6$ slices of pepperoni pizza, so $5x + 6(x + 0.5) = 11x + 3 = 25$. Since $x = 2$ from the previous equation, a slice of plain pizza costs \$2 and a slice of pepperoni pizza costs \$2.50. David ordered 3 slices of plain pizza and 2 slices of pepperoni pizza, so he paid $3 \cdot 2 + 2 \cdot 2.5 = 6 + 5 = \boxed{11}$ dollars.

27. There are 60 seconds in a minute, so his song is $3 \cdot 60 + 45 = 225$ seconds long. There are 60 minutes in an hour, so there are $60 \cdot 60 = 3600$ seconds in an hour. Thus the final fraction is $\frac{225}{3600} = \boxed{\frac{1}{16}}$.

28. Using the formula for average , we have $\frac{34+35+39}{3} - \frac{23-14-17}{3} = \frac{34+35+39-23-14-17}{3} = \boxed{18}$.

29. 54 flips $= 18 \cdot 5 = 90$ flops. 90 flops $= 10 \cdot 14 = \boxed{140}$ flaps. The LCM must divide all of the numbers so prime factorizing each of them first gives 8 as 2^3, 78 as $2 \cdot 3 \cdot 13$ and 130 as $2 \cdot 5 \cdot 13$. Then to get the LCM to be divisible by all three numbers we multiply $2^3 \cdot 3 \cdot 5 \cdot 13$ which is $\boxed{1560}$.

30. There is a probability of $\frac{1}{3}$ of pulling out the letter B first; then a probability of $\frac{1}{2}$ of pulling out C; the letter A then has a $\frac{1}{1}$ chance of being selected. Multiplying the fractions together gives a total probability of $\boxed{\frac{1}{6}}$.

31. Andrew and Dennis finish 3% in 1 hour, so the other three will do 15% in one hour. There is 97% left, so the answer is simply $\frac{97}{15} \cdot 60 = \boxed{388}$.

32. Arthur ran 40 meters in the first 5 seconds. He only has to run for $\frac{100-40}{3} = \frac{60}{3} = 20$ more seconds. Sunny ran for 32 meters in the first 8 seconds. That means that in $25 - 8 = 17$ seconds, he must run 68 meters, which is an average speed of $\frac{68}{17} = \boxed{4}$ m/s.

33. On the nth day, Ryan has cleared $5n$ levels and Max has cleared $\frac{n(n+1)}{2}$ levels. If it takes k days for Max to catch up to Ryan, $5k = \frac{k(k+1)}{2}$. Manipulating the prior equation, $k^2 - 9k = k(k-9) = 0$. There are two solutions, $k = 0, 9$. Since the question is asking how many days it will take after they both start playing, the answer is $\boxed{9}$ days.

34. The number of permutations disregarding the repeated alphabat is $5! = 120$. However, the letter M is repeated twice, thus the number should be divided by $2!$, yielding $\boxed{60}$ as the answer.

35. Let k be a positive integer, and let $k = (n\#3)\#3$. Then $3k = (n\#3) - 2$, or $3k + 2 = \frac{n-2}{3}$ so $9k + 6 = n - 2$ and $n = 9k + 8$. We see that n is minimal when k is minimal, so letting $k = 1$ we find the smallest positive integer value is $n = \boxed{17}$.

36. The ratio of the areas is $\frac{360}{40} = 9$, hence the ratio of the sides will be $\sqrt{9} = 3$. The length h of the larger hypotenuse will satisfy $\frac{h}{15} = 3$, and we find $h = \boxed{45}$.

37. Since $\frac{x}{x+2} < \frac{61}{64}$, multiplying both sides of the inequality by $64(x+2)$ yields $64x < 61(x+2)$, which can be simplified to $3x < 122$, then the largest integer value for x would be $\boxed{40}$.

38. Let the distance between their houses be x, Haneul's speed in meters per second be H, and Julia's speed in meters per second be J. To the first meeting point, Haneul travelled 200 meters while Julia travelled $x - 200$ meters. So, since the time was the same, $\frac{J}{H} = \frac{x-200}{200}$. Let's assume $H = 1$, so $J = \frac{x-200}{200}$. To the second meeting point, Haneul travelled $x + 100$ meters and Julia travelled $x + (x - 100) = 2x - 100$ meters. Again, since the time wa the same, $\frac{J}{H} = \frac{2x-100}{x+100}$. Since $H = 1$, we have $J = \frac{2x-100}{x+100}$. Putting our two equations together, we get $\frac{x-200}{200} = \frac{2x-100}{x+100}$, or $(x-200)(x+100) = (2x-100)(200)$, or $x^2 - 100x - 20000 = 400x - 20000$, or $x^2 = 500x$, or $x = \boxed{500}$.

39. The net rate of water flow into the tank is $4 - 2.5 = 1.5$ liters per minute. Thus, it will take $\frac{150}{1.5} = 100$ minutes to fill the tank, or, equivalently, $\boxed{6000}$ seconds.

40. From the given averages, we have $a+b+c+d = 28$ and $d+e+f+g = 16$. We also have $a+b+c+d+e+f+g = 35$. Adding the first two equations we have $a+b+c+2d+e+f+g = 44$, and subtracting the third from this gives us $d = \boxed{9}$.

41. In the first 2 days, exactly 20% of the first wall is built. That leaves 4.8 walls left. Typically, it would take 10 workers 48 days to build 4.8 walls. Since there are only 8 days left, the workers must do 6 times as much to get the same work done in $\frac{8}{48} = \frac{1}{6}$ of the time. The workers can not work faster, so that means there must be 6 times as many workers, making the final amount 60. We are looking for the amount added, so the answer will thus be $60 - 10 = \boxed{50}$.

42. Let's call our three non-negative integers x, y, z. We are given that $x + y = 10 - z$, or $x + y + z = 10$. This can be translated into a stars-and-bars problem, with 10 stars and 2 bars. As a result, our answer is $\binom{12}{2} = \frac{12 \cdot 11}{2} = \boxed{66}$.

43. The third side must be either 32 or 65, but by the triangle inequality it cannot be 32, so we have a $65, 65, 32$ triangle. We want to find the length of the altitude to the base, call it h. Then we have $16^2 + h^2 = 65^2$, or $h^2 = 65^2 - 16^2 = (65-16)(65+16) = (7^2)(9^2) = 63^2$, so $h = 63$ and our area is $\frac{32 \cdot 63}{2} = 16 \cdot 63 = \boxed{1008}$.

44. In a round robin with n teams, the number of games played is essentially summing 1 through $n - 1$, which yields $\frac{n(n-1)}{2} = 90$. Solving for n yields $\boxed{10}$ as the answer.

45. We can apply the Euclidean algorithm to this problem to eventually obtain that $\gcd(5x+18, 8x+30)$ is equal to $\gcd(6, x)$. Thus, the number of possibilities for the GCD is just the number of positive divisors of 6, so our answer is $\boxed{4}$.

46. In a standard 8 by 8 chess board, there are 9 horizontal lines and 9 vertical lines, counting the borders. A rectangle is made of 2 vertical lines and 2 horizontal lines. There are $\binom{9}{2} = 36$ ways to select 2 horizontal lines, and $\binom{9}{2} = 36$ ways to select 2 vertical lines. These are independent events, so multiplying gives us $36^2 = \boxed{1296}$.

47. The average angle in a convex 12-gon is $180(12-2)/12 = 150$ degrees. Since there is an even number of terms in the arithmetic sequence, the middle 2 terms differ from 150 by an integer d. These middle terms can be written as $150+d$ and $150-d$, and each consecutive term in the sequence differs by 2d. The largest term in the sequence is $(150+d)+10d = 150+11d$. Because the polygon is convex, $150+11d < 180$. To minimize the smallest angle, we want to maximize d, so d = 2. The smallest angle is $150 - 11d = 150 - 22 = \boxed{128}$.

48. We find the sum of the reciprocals of all odd squares by subtracting the sum of reciprocals of all even squares from the total sum of reciprocals of all squares and rewriting the sum of the even-square reciprocals in terms of the total sum. Mathematically, this looks like

$$\begin{aligned}
\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots &= \left(\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots\right) - \left(\frac{1}{2^2}+\frac{1}{4^2}+\frac{1}{6^2}+\cdots\right) \\
&= \left(\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots\right) - \frac{1}{2^2}\left(\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots\right) \\
&= \left(1-\frac{1}{2^2}\right)(S) \\
&= \boxed{\frac{3S}{4}}
\end{aligned}$$

49. Let the probability that David becomes broke be p. After two days, Hannah can become broke, David can become broke, or they can return to their original distribution with two drips each. David becomes broke in two days with probability $(\frac{1}{4})^2 = \frac{1}{16}$, and they return to their original distribution with probability $\frac{3}{4}\cdot\frac{1}{4}+\frac{1}{4}\cdot\frac{3}{4} = \frac{3}{8}$. Thus, $p = \frac{1}{16}+\frac{3}{8}p$. Multiplying both sides by 16 and rearranging gives us $p = \boxed{\frac{1}{10}}$, and we are done.