

# Joe Holbrook Memorial Math Competition

4th Grade

October 15, 2017

## General Rules

- You will have **75 minutes** to solve **40 questions**. Your score is the number of correct answers.
- Only answers recorded on the answer sheet will be graded.
- This is an individual test. Anyone caught communicating with another student will be removed from the exam.
- Scores will be posted on the website. Please do not forget your ID number, as that will be the sole means of identification for the scores.
- You may not use the following aids:
  - Calculator or other computing device
  - Compass
  - Protractor
  - Ruler or straightedge

In addition, you must use the scrap paper supplied by the proctors.

## Other Notes

- Write legibly. If the graders cannot read your answer, you will be given no credit for that question.
- Fractions should be written in lowest terms. Please convert all mixed numbers into improper fractions.
- For constants such as  $e$  or  $\pi$ , do not approximate your answer: for example, if the answer to a question is  $7\pi$ , then you should not write 22 or 21.99.
- You do not need to write units in your answers.
- Rationalize all denominators. In addition, numbers within a square root must be squarefree, e.g.  $\sqrt{63}$  should be written as  $3\sqrt{7}$ .
- Ties will be broken by the number of correct responses to questions 31 through 40. Further ties will be broken by the number of correct responses in the last five questions.

1. On average, how many minutes should be spent per problem in this contest?
2. Youjung loves playing golf. She played total of 9 holes and scored 2 3's, 5 4's, 2 5's. What is her total score?
3. It is currently 12:24pm and Jonathan's mom won't let him go swimming until 3:00pm (on the same day). How many minutes does he have to wait until he can go?
4. Adam is driving a car, and takes 1 minute and 7 seconds to travel 1 mile. How long does it take to travel 60 miles? Express your answer in minutes.
5. In Reverse Land, addition and subtraction come before division and multiplication, and operations are performed from right to left. What is  $2 + 5 \cdot 2 - 5$  in Reverse Land?
6.  $ABCD$  is a square with side length 6. Let  $E$  be the midpoint of  $BC$ . If  $F$  and  $G$  are points outside  $ABCD$  such that  $EFGC$  is a square, find the perimeter of  $ABEFGCD$ .
7. What is the largest prime factor of 2670?
8. BCA Math Team loves donuts. At the last practice, there were 300 donuts. 20% of the donuts had sprinkles, 10% of the donuts had chocolate, and 37% of the donuts had jelly. The rest were plain. How many plain donuts were there?
9. Bill's phone has 40% battery left, so he decides to start charging it. The battery charges at 2% per minute if he is not using it. But, he starts using his phone, which discharges it at 0.5% per minute. How long will it take for his battery to charge fully?
10. It takes Malik 12.21 seconds to run 100 meters. However, he is the greatest distance runner of all time and his speed remains constant no matter how tired he gets. How many seconds does it take Malik to complete a 5 kilometer race?
11. 12 students line up in a classroom. The teacher tells every second student to raise their right hand. Then, the teacher tells every third student to raise their left hand. How many students have both hands raised?
12. What is  $\frac{3}{4} + \frac{4}{5} + \frac{5}{3}$ ? Express your answer as a common fraction.
13. Jaylen is trying to improve his basketball shooting percentage at the gym, and he refuses to go home until he has made at least 70% of the shots he has taken. So far, he has taken 15 shots and made only 7 of them. Assuming that from now on, he never misses a shot, what is the minimum number of shots that he has to make consecutively to reach his goal?
14. Jaylen and Derrick want to play basketball together, so they agree to meet up at a local park that is 2 miles away from both of their houses. Derrick rides his skateboard to the park at a constant rate of 5 miles per hour, while Jaylen walks to the park at a constant rate of 3 miles per hour. If the boys want to arrive at the park at the same time, how many minutes earlier than Derrick should Jaylen leave his house?
15. Pete is on a road trip with his wife. He spends  $\frac{3}{8}$  of the trip on cruise control,  $\frac{1}{3}$  of the trip actively driving, and lets his wife take over for the remaining 840 miles. How many miles does he drive with cruise control enabled?
16. The entire BCA Math Team are trying to cross the Hackensack River and we have a bunch of boats. We realize that if we have another boat, there will be exactly 6 people on each boat; if we take away a boat, there will be exactly 9 people on each boat. How many people are on the team?
17. A BCA student is choosing classes for junior year. There are three choices for literature class, two choices for history class, four choices for math class, and two choices for foreign language class. In how many ways can the courses be chosen, if exactly one class is chosen from each category?
18. Wanye the Toed is creating the Council of Toed from him and his friends, Peiyu the Pig, Greggy the Goose, Sampai the Snail, Daniel the Stewart, and Andrew the Antelope. The Council should have 3 members, and Wanye has to be in it. How many choices are there for the Council?
19. Two circles have radius 3 and 5. Their centers are 14 apart. A line segment connects one point on one circle to another point on the other circle. What is the minimum length of all possible line segments?

20. Julia and friends are in a yellow submarine. They see octopuses (8 legs) and crabs (10 legs). In total, they see 256 legs and 29 heads. How many octopuses are there?
21. Haneul and Julia are at opposite ends of the country. In order to communicate regularly, they agree to video-call each other from 4:00pm to 4:30pm on every perfect square date of the month, and to talk on the phone from 5:00pm to 6:00pm on any day that is a perfect power of 2 (if they did not already video-call on this day). How many hours do they spend communicating in the month of July?
22. When I add 39 to a number, the result is 5 less than 2 times the original number. What is the original number?
23. Youjung and Yousun have 48 pieces of candy in total. If Yousun gives Youjung 5 candies to Youjung, they would have same number of candies. How many candies did Yousun have originally?
24. Because of the JHMMC, all stores in New Jersey are having a sale. A jacket that was \$12 is now on sale at a 10% discount. Assuming the sales tax is 10%, what is the final price of the jacket?
25. What is the unit digits of  $2017^{2017}$ ?
26. Annie mows the lawn every 6 days. If the first time she mowed the lawn this year was on a Sunday, what day of the week will it be when she mows her lawn for the 30th time?
27. Find all integers  $n$  which satisfy the inequality  $n^2 - 3n + 2 \leq 0$ .
28. An evil goat spelled the name *Tifany* wrong. The goat had written her name as *Tiffany*. How many more ways are there to rearrange the letters of *Tiffany* than *Tifany*?
29. If the least common multiple of positive integers  $a, b$  is 20 and the greatest common divisor of  $a, b$  is 10, find the product  $a \cdot b$ .
30. Let  $a \Delta b = \frac{2ab}{a+b}$ . Find  $((2\Delta 6)\Delta 3)\Delta(0\Delta(3\Delta 7))$ .
31. In the world of Tri, the inhabitants have three feet. Trigo, an inhabitant of Tri, has 6 red socks, 6 blue socks, and 6 green socks. In the early morning, Trigo grabs 3 random socks and puts them on without looking. What is the probability that all three are of the same color?
32. Rhombus  $ABCD$  has side length 6 and  $\angle ABC = 120^\circ$ . What is the area of rhombus  $ABCD$ ?
33. David is standing 6 feet away from a tree. He sees the top of the tree at an angle of 60 degrees to the ground. If David spots a bug crawling along the tree at an angle of 30 degrees to the ground, how far above the ground was the bug at that instant? (Note: David's height is negligible)
34. How many of the factors of 1200 are not perfect squares?
35. Let  $ABCD$  be a rectangle with  $AB = 8$  and  $BC = 6$ . If  $M$  is the midpoint of  $BD$  and  $N$  is the foot of the perpendicular from  $A$  to  $BD$ , find  $MN$ .
36. Parallelogram  $ABCD$  has a perimeter of 75. When side  $BC$  is the base, the height is 14. When side  $CD$  is the base, the height is 16. Find the area of parallelogram  $ABCD$ .
37. How many positive integers divide at least two of the numbers 84, 120, and 126?
38. Find the number of zeros at the end of  $2017!$  when written in base 3.
39. What is the probability that when 3 edges are chosen from a cube, they are pairwise skew? Two lines are skew if they are not parallel and do not intersect each other.
40. The numbers 0, 0, 1, 1, 2, 2, 7, and 7 are arranged in a sequence. An entry in this sequence is called pure if there is no number after it in the sequence that is greater than it (for example, in the sequence 0, 1, 7, 2, 7, 2, 1, 0, the right 2 in the arrangement is pure). How many sequences are there such that at least one of the 2's is pure?