Joe Holbrook Memorial Math Competition

6th Grade

October 15, 2017

General Rules

- You will have **75 minutes** to solve **40 questions**. Your score is the number of correct answers.

- Only answers recorded on the answer sheet will be graded.

- This is an individual test. Anyone caught communicating with another student will be removed from the exam.

- Scores will be posted on the website. Please do not forget your ID number, as that will be the sole means of identification for the scores.

- You may not use the following aids:
  - Calculator or other computing device
  - Compass
  - Protractor
  - Ruler or straightedge

  In addition, you must use the scrap paper supplied by the proctors.

Other Notes

- Write legibly. If the graders cannot read your answer, you will be given no credit for that question.

- Fractions should be written in lowest terms. Please convert all mixed numbers into improper fractions.

- For constants such as e or π, do not approximate your answer: for example, if the answer to a question is 7π, then you should not write 22 or 21.99.

- You do not need to write units in your answers.

- Rationalize all denominators. In addition, numbers within a square root must be squarefree, e.g. $\sqrt{63}$ should be written as $3\sqrt{7}$.

- Ties will be broken by the number of correct responses to questions 31 through 40. Further ties will be broken by the number of correct responses in the last five questions.

1. It is currently 12:24PM and Jonathan's mom won't let him go swimming until 3:00PM (on the same day). How many minutes does he have to wait until he can go?

2. Adam is driving a car, and takes 1 minute and 7 seconds to travel 1 mile. How long does it take to travel 60 miles? Express your answer in minutes.

3. In Reverse Land, addition and subtraction come before division and multiplication, and operations are performed from right to left. What is $2 + 5 \cdot 2 - 5$ in Reverse Land?

4. $ABCD$ is a square with side length 6. Let E be the midpoint of BC. If F and G are points outside $ABCD$ such that $EFGC$ is a square, find the perimeter of $ABEFGCD$.

5. BCA Math Team loves donuts. At the last practice, there were 300 donuts. 20% of the donuts had sprinkles, 10% of the donuts had chocolate, and 37% of the donuts had jelly. The rest were plain. How many plain donuts were there?

6. Jaylen and Derrick want to play basketball together, so they agree to meet up at a local park that is 2 miles away from both of their houses. Derrick rides his skateboard to the park at a constant rate of 5 miles per hour, while Jaylen walks to the park at a constant rate of 3 miles per hour. If the boys want to arrive at the park at the same time, how many minutes earlier than Derrick should Jaylen leave his house?

7. The Big Clock of America flashes every 9 minutes and rings every hour. The last time the clock both flashed and rang was at 12:00 PM. When will be the next time that the clock both flashes and rings? Indicate AM/PM.

8. The entire BCA Math Team are trying to cross the Hackensack River and we have a bunch of boats. We realize that if we have another boat, there will be exactly 6 people on each boat; if we take away a boat, there will be exactly 9 people on each boat. How many people are on the team?

9. Wanye the Toed is creating the Council of Toed from him and his friends, Peiyu the Pig, Greggy the Goose, Sampai the Snail, Daniel the Stewart, and Andrew the Antelope. The Council should have 3 members, and Wanye has to be in it. How many choices are there for the Council?

10. How many whole numbers are between $\sqrt{7}$ and $\sqrt{700}$?

11. Because of JHMMC, all stores in New Jersey are having a sale. A jacket that was \$12 is now on sale at a 10% discount. Assuming the sales tax is 10%, what is the final price of the jacket?

12. Trevon averaged 25 points per game over his last 7 basketball games. In the first four of these, he averaged only 14 points per game. However, he averaged 31 points per game over the last four games. How many points did he score in his 4th game?

13. What is the unit digits of 2017^{2017}?

14. Find all integers n which satisfy the inequality $n^2 - 3n + 2 \leq 0$.

15. Sarah is making a sandwich for lunch. In her fridge, she has 5 types of meat: ham, salami, turkey, baloney, and roast beef, as well as 3 types of cheese: swiss, american, and provolone. She wants her sandwich to have 2 different types of meat and 1 type of cheese, but absolutely hates the taste of salami with american cheese or roast beef with baloney. How make types of sandwiches can Sarah make for lunch that she will actually like?

16. David Ni likes to hug cats and dogs on his birthday. It costs \$2 dollars to hug a dog once and \$10 to hug a cat once. He spends \$200 on his birthday hugging cats and dogs. He hugged the cats and dogs a total of 32 times. How many times did David Ni hug a cat?

17. Let $a\Delta b = \dfrac{2ab}{a+b}$. Find $((2\Delta 6)\Delta 3)\Delta(0\Delta(3\Delta 7))$.

18. The Fibonacci sequence is a sequence of numbers where any term after the second is the sum of the previous two terms. Among the first 100 terms of the sequence, how many are even? The first 8 terms are given here: 1, 1, 2, 3, 5, 8, 13, 21.

19. The average, median, and unique mode of 5 positive numbers is 5. What is the maximum possible value of the largest number in those positive numbers?

20. What is the maximum value of $a^b \cdot c^d$, where $\{a, b, c, d\}$ is some permutation of $\{2, 0, 1, 7\}$?

21. Rhombus ABCD has side length 6 and $\angle ABC = 120°$. What is the area of rhombus ABCD?

22. The difference of two 3-digit numbers is 894. Find the product of the digits of the 2 numbers.

23. Ben, David, Haneul, Ivy, and Julia go to the movies. Julia does not want to sit next to neither Ben nor David and Ivy insists that she sits next to Haneul. How many ways are there for the five friends to sit?

24. If a competitor has a chance of $\pi\%$ chance of flipping each number backwards (e.x. 421 into 124) in addition problems, what is the probability that this competitor will find that $3759 + 462 + 37145 + 12334$ is a multiple of 3?

25. David is standing 6 feet away from a tree. He sees the top of the tree at an angle of 60 degrees to the ground. If David spots a bug crawling along the tree at an angle of 30 degrees to the ground, how far above the ground was the bug at that instant? (Note: David's height is negligible)

26. Triangle ABC has circumcircle Ω with center O and radius 2. If line BO meets Ω again at a point P other than A with $PC = 3$, find the length of side BC.

27. How many of the factors of 1200 are not perfect squares?

28. Let $ABCD$ be a rectangle with $AB = 8$ and $BC = 6$. If M is the midpoint of BD and N is the foot of the perpendicular from A to BD, find MN.

29. Parallelogram $ABCD$ has a perimeter of 75. When side BC is the base, the height is 14. When side CD is the base, the height is 16. Find the area of parallelogram $ABCD$.

30. Ledward got confused when trying to build his stable in a grassy meadow and ended up building it in the shape of an equilateral triangle with side length 6 meters. Boopkhun visited Ledward and tied his horse to a vertex of the barn with a rope of length 8 meters. What is the area of meadow that Boopkhun's horse can graze in?

31. A group of 10 people are taking a math test, but one of them is cheating! If the 5 proctors each pick one different person to kick out, what is the probability that the cheater will be caught?

32. It takes 4 people, 8 dolphins, 40 chickens, or 50 rats to screw in a light bulb in 1 hour, Assuming that each animal can work together without loss of productivity, how many rats are needed along with a human, 5 dolphins, and 25 chickens to screw in a light bulb in 30 minutes?

33. Let a and b be two distinct real numbers. If the function $f(x) = x^2 + ax + b$ satisfies $f(a) = f(b)$, find the value of $f(2)$.

34. Let n be the third smallest positive integer that satisfies the following conditions: It leaves a remainder of 5 when divided by 7, and it leaves a remainder of 4 when divided by 9. Find n.

35. An ant is at $(0,0)$ on the coordinate plane and wants to get to $(5,7)$, It can only move up or right. How many routes can the ant take to get to its destination if it cannot pass through $(1,3)$ or $(3,1)$?

36. How many solutions are there to the equation $x + y + z = \sqrt{2xy + 2yz + 2xz + 41}$ where x, y, and z are nonnegative integers?

37. If x satisfies $x^2 + x + 1 = 0$, compute $x^{200} + x^{100} + 1$.

38. In trapezoid $ABCD$, $AB \parallel CD$, $AC = 11, BD = 60$, and $AC \perp BD$. If M and N are the midpoints of AD and BC, respectively, what is the length of segment MN?

39. Two trains are running at a right angle from each other towards the same intersection. The train running north is running at 50 mph, is 200 miles from the intersection, and is 25 miles long. The train running east is 300 miles from the intersection, and is 50 miles long. What is the difference between the minimum and maximum speed that the train running east can run such that the two trains will collide?

40. Given that $\dfrac{2022!}{2017!} = 33,632,280,3AB,168,080$, where A and B are digits, what is $10A + B$?