

Joe Holbrook Memorial Math Competition

4th Grade

October 14, 2018

General Rules

- You will have **75 minutes** to solve **40 questions**. Your score is the number of correct answers.
- Only answers recorded on the answer sheet will be graded.
- This is an individual test. Anyone caught communicating with another student will be removed from the exam.
- Scores will be posted on the website. Please do not forget your ID number, as that will be the sole means of identification for the scores.
- You may not use the following aids:
 - Calculator or other computing device
 - Compass
 - Protractor
 - Ruler or straightedge

In addition, you must use the scrap paper supplied by the proctors.

Other Notes

- Write legibly. If the graders cannot read your answer, you will be given no credit for that question.
- Fractions should be written in lowest terms. Please convert all mixed numbers into improper fractions.
- For constants such as e or π , do not approximate your answer: for example, if the answer to a question is 7π , then you should not write 22 or 21.99.
- You do not need to write units in your answers.
- Rationalize all denominators. In addition, numbers within a square root must be squarefree, e.g. $\sqrt{63}$ should be written as $3\sqrt{7}$.
- Ties will be broken by the number of correct responses to questions 31 through 40. Further ties will be broken by the number of correct responses in the last five questions.

- Members of the BCA Math Team are writing problems. If they write 5 problems every minute, how long, in minutes, will it take for them to write 80 problems?
- What is the value of $2 \div (0 + 1) \cdot 8$?
- What fraction of the months in a year have 31 days?
- After Shalin got his driver's license, his friends gave him a chocolate cake. First, Shalin ate $\frac{1}{3}$ of it. Then, Jenn ate $\frac{1}{2}$ of what remained. What fraction of the cake was left?
- A quarter of all students taking the JHMMC registered in August. If 100 students registered in August, how many total students are taking the JHMMC?
- Lunch at the Bergen County Academies cafeteria costs four dollars. There are ten months in the school year, with eighteen school days each. Joe buys lunch (and nothing else) at the Bergen County Academies cafeteria on every school day. How much money, in dollars, will he spend over one entire school year?
- Compute $1003 + 130 + 310 + 3001$.
- Compute $2018 + 20.18 + 0.2018$.
- What decimal lies halfway between $\frac{1}{2}$ and $\frac{1}{4}$?
- Every day, a student solves three problems more than she did the day before. If she solved four problems on the first day, how many problems did she solve on the twelfth day?
- Alan is watching a soccer tournament. There are seven teams playing, and each team plays exactly once against every other team. How many games are played in total?
- Allison rolls a fair 6-sided die. What is the probability that she rolls a prime number?
- Haneul the Beaver wants to be tall. She is 35 inches tall. She stands on a ledge that is 1 foot and 2 inches tall. Her friend Andrew the Sun Bear is 4.5 feet tall. How many inches shorter is she compared to Andrew while she is standing on the ledge?
- Mikako the Monkey has a favorite positive number. She multiplies this number by 4 and then subtracts 11. Finally, she takes the square root of the result to get 3. What is Mikako's favorite positive number?
- A hydra starts out with 20 heads, and grows 3 new heads every time one of its heads is chopped off. How many heads will the hydra have if 7 heads are chopped off, one at a time? (Hydra heads grow instantly.)
- Kelvin the Frog wants to buy 100 \$1 McDonald's chicken nuggets. Right now, he has \$25 and he earns \$4 an hour from his job. He only works a whole number of hours. How many more hours does he have to work in order to afford the 100 chicken nuggets?
- In the first minute, Herb makes 7 pizzas, and then Anna eats 3. In each subsequent minute, he makes another 7, and then Anna eats another 3. During which minute does the pizza count exceed 2018 for the first time?
- What is the positive difference between the sum of factors of 2018 and the number of factors of 2019?
- To get to school from home, Jason walks 8 meters west then 15 meters south. How many fewer meters would Jason walk if he walked in a straight line from home to school?
- In how many ways can 2019 stones be split into two piles each with a prime number of stones such that the pile on the left has less stones than the pile on the right?
- Gregory, a monster, gives math problems to children in his free time. However, he does not give math problems to any children who are two years old or younger. He will give math problems to any other child he encounters. One day, he encountered ten children of the following ages: 1, 1, 2, 3, 4, 7, 8, 9, 11, and 12. Determine the median age of the children he gives math problems to on that day.
- If $a \star b = 5a + 3b + (b \star (a - 1))$ and $1 \star 1 = 10$, then what is $2 \star 2$?
- A bag is filled with 600 Starbursts, 30% of which are pink. How many pink Starbursts must be removed from the bag for 20% of the remaining Starbursts to be pink?

24. How many digits are in the base-10 representation of 1234567^2 ?
25. Alan's calculator only has the operations of multiplying by 2 and adding 1 to the number on the screen. His calculator starts with the number 2. What is the minimum number of operations that Alan has to use to get to the number 35?
26. An infinite line of dominoes labeled $1, 2, 3, \dots$ is placed in a line in that order. The domino labeled 1 is knocked over with probability 1. For $n \geq 2$, if the domino labeled $n - 1$ is knocked over, then the domino labeled n gets knocked over with probability $\frac{n}{n+1}$. What is the probability that the domino labeled 2018 will be knocked over?
27. Andrew the Sun Bear has taken five 100-point math tests. His average score on these five tests is 94. What minimum score must Andrew get on his sixth math test in order to raise his average to 95?
28. There are several positive integers x such that $\text{lcm}(4, x) = 100$. What is the sum of all possible values of x ?
29. Andrew the Sun Bear is writing problems for the Joe Holbrook. Though he enjoys writing problems for younger students, he grows increasingly sleepy. Therefore, the rate at which his brain can produce interesting questions decreases over time. In fact, Andrew's younger brother, Simon, notices a pattern: Andrew's first problem takes him one minute to write, his second problem takes him two minutes to write, his third problem takes him three minutes to write, and so on. Assuming that Andrew does not take any breaks, how long does it take him to write his first ten problems?
30. Sort the numbers in $\{4, 8, 12, 6, 9, 3\}$ such that to get from each number to the next, one must either multiply by 2 or divide by 3. Your answer should be an ordered six-tuple of the form (a, b, c, d, e, f) .
31. If exactly $\frac{5}{8}$ of the people in a room like chocolate ice cream, and if $\frac{3}{5}$ of the people in the room like vanilla, then what is the smallest possible number of people in the room who like both chocolate and vanilla ice cream?
32. An equilateral triangle has side-length 20. A square has side-length 13. Determine the area of the figure whose area is greater.
33. At the grocery store, Sebastian only bought apples, which have a price of some whole number of cents. He bought at least two apples but strictly less than 30 apples. If Sebastian paid the cashier \$10 and received \$4.61 in change, what is the sum of all possible values for the number of apples Sebastian bought?
34. There are 5 distinguishable children at a party. Susan has 7 identical candies to give out to these children. If each child must receive at least one candy, how many ways can Susan give out her candies?
35. A lamb is tied to a post located at the origin of the coordinate plane by a rope that measures 6 units. The farmer who owns the lamb also keeps two wolves tied up at $(6, 6)$ and $(-6, -6)$ with ropes that measure 6 units. What is the area of the region that the lamb can wander in without being in the range of the wolves? (Note: all motion is restricted to the coordinate plane.)
36. Let the roots of the polynomial $(x - 5)(x - 7)$ be p and q . What is the value of $\frac{p}{q} + \frac{q}{p}$?
37. Two trains, 360 miles apart, begin moving directly towards each other at 60 miles per hour. A crow, flying at 40 miles per hour, is flying between the trains. It begins midway between the trains and starts flying toward one of the trains. Every time the crow encounters a train, it reverses its direction while maintaining its speed. When the trains meet, what total distance has the crow traveled?
38. Let ϕ denote the number of yards to Alpha Centauri. What is the minimum value of

$$\frac{x^2}{2018} + \frac{\phi^2 + 2017}{2018} + \frac{\phi x}{1009},$$

as x varies over the real numbers?

39. How many ways are there to rearrange the letters of "seventeen" so that the consonants are either in alphabetical order or in reverse alphabetical order?
40. Compute the sum

$$\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \dots + \frac{1}{17 \cdot 19}.$$