# Joe Holbrook Memorial Math Competition

5th Grade

October 14, 2018

**General Rules**

- You will have **75 minutes** to solve **40 questions**. Your score is the number of correct answers.
- Only answers recorded on the answer sheet will be graded.
- This is an individual test. Anyone caught communicating with another student will be removed from the exam.
- Scores will be posted on the website. Please do not forget your ID number, as that will be the sole means of identification for the scores.
- You may not use the following aids:
  - Calculator or other computing device
  - Compass
  - Protractor
  - Ruler or straightedge

  In addition, you must use the scrap paper supplied by the proctors.

**Other Notes**

- Write legibly. If the graders cannot read your answer, you will be given no credit for that question.
- Fractions should be written in lowest terms. Please convert all mixed numbers into improper fractions.
- For constants such as $e$ or $\pi$, do not approximate your answer: for example, if the answer to a question is $7\pi$, then you should not write 22 or 21.99.
- You do not need to write units in your answers.
- Rationalize all denominators. In addition, numbers within a square root must be squarefree, e.g. $\sqrt{63}$ should be written as $3\sqrt{7}$.
- Ties will be broken by the number of correct responses to questions 31 through 40. Further ties will be broken by the number of correct responses in the last five questions.

1. Members of the BCA Math Team are writing problems. If they write 5 problems every minute, how long, in minutes, will it take for them to write 80 problems?
2. What is the value of $2 \div (0+1) \cdot 8$?
3. What fraction of the months in a year have 31 days?
4. After Shalin got his driver's license, his friends gave him a chocolate cake. First, Shalin ate $\frac{1}{3}$ of it. Then, Jenn ate $\frac{2}{5}$ of what remained. What fraction of the cake was left?
5. A quarter of all students taking the JHMMC registered in August. If 130 students registered in August, how many total students are taking the JHMMC?
6. Lunch at the Bergen County Academies cafeteria costs four dollars. There are ten months in the school year, with eighteen school days each. Joe buys lunch (and nothing else) at the Bergen County Academies cafeteria on every school day. How much money, in dollars, will he spend over one entire school year?
7. Compute $1009 + 190 + 910 + 9001$.
8. What number lies halfway between $\frac{1}{2}$ and $\frac{1}{4}$?
9. Every day, a student solves three problems more than she did the day before. If she solved four problems on the first day, how many problems did she solve on the twelfth day?
10. Mikako the Monkey has a favorite positive number. She multiplies this number by 4 and then subtracts 11. Finally, she takes the square root of the result to get 3. What is Mikako's favorite positive number?
11. A hydra starts out with 20 heads, and grows 3 new heads every time one of its heads is chopped off. How many heads will the hydra have if 7 heads are chopped off one at a time?
12. Kelvin the Frog wants to buy 100 \$1 McDonald's chicken nuggets. Right now, he has \$25. He has a job from which he earns \$4 an hour, and he only works a whole number of hours. How many more hours does he have to work in order to afford the 100 chicken nuggets?
13. Herb eats $\frac{1}{4}$ of a pizza and Anna eats half of the remaining pizza. How much of the pizza is left?
14. What is the positive difference between the sum of the factors of 2018 and the number of factors of 2019?
15. In how many ways can 2019 be written as the sum of two prime numbers (where order does not matter)?
16. Autumn is stuck in traffic because there are cows crossing the road! The cows cross the road one at a time and a different cow crosses immediately after the one in front of it. Adult cows take 2 minutes to cross, but baby cows take 5 minutes. If Autumn had to wait exactly 38 minutes for all the cows to cross and she saw exactly 10 cows while she was waiting, how many baby cows did she see (each cow is either an adult or baby)?
17. How many digits are in the base-10 representation of $1234567^2$?
18. Alan's calculator only has the operations of multiplying by 2 and adding 1 to the number on the screen. His calculator starts with the number 2. What is the minimum number of operations that Alan has to use to get to the number 35?
19. An infinite line of dominoes labeled $1, 2, 3\ldots$ is placed in a line in that order. The domino labeled 1 is knocked over with probability 1. For $n \geq 2$, if the domino labeled $n-1$ is knocked over, then the domino labeled $n$ gets knocked over with probability $\frac{n}{n+1}$. What is the probability that the domino labeled 2018 will be knocked over?
20. Andrew the Sun Bear has taken five 100-point math tests. His average score on these five tests is 94. What minimum score must Andrew get on his sixth math test in order to raise his average to 95?
21. Let $x$ be a positive integer so that $\text{lcm}(4, x) = 100$. What is the sum of all possible values of $x$?
22. At a dance, there are 120 people. Some are paired up so that there are 48 couples. What is the probability that a randomly selected person is not in a couple?

23. Oh, no! The right half of a standard 2-hand 12-hour analog clock is blocked! For how many hours in a 24-hour day is at least one of the hands not visible?

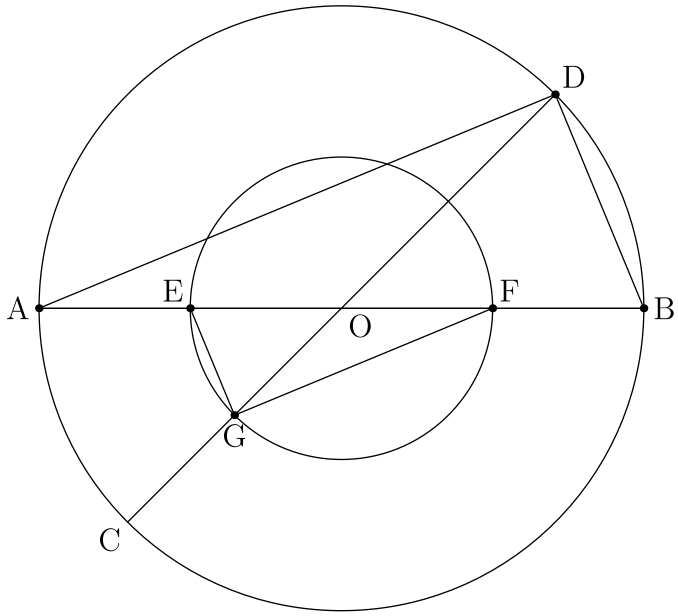
24. Sort the numbers in $\{4, 8, 12, 6, 9, 3\}$ such that to get from each number to the next, one must either multiply by 2 or divide by 3. Your answer should be an ordered six-tuple of the form $(a, b, c, d, e, f)$.

25. A monarch is being served food, which comes in the form of a row of 4 pieces of chocolate and 4 pieces of steak, in some order. However, the server is a greedy man, and will take pieces of chocolates for himself and replace them with pieces of kale. Assuming the server can choose as many pieces of chocolate as he wants (or none at all), how many possible orders of food can the monarch receive?

26. A lamb is tied to a post located at the origin of the coordinate plane by a rope that measures 6 units. The farmer who owns the lamb also keeps two wolves tied up at $(6, 6)$ and $(-6, -6)$ with ropes that measure 6 units. What is the area of the region that the lamb can wander in without being in the range of the wolves? (Note: all motion is restricted to the coordinate plane.)

27. Triangle $ABC$ has altitude $AH$ with $H$ lying on $BC$. Given that $AB = 13$, $AH = 12$, and $BC = 14$, find $AC$.

28. Two circles are centered at point $O$, one with radius 3 and one with radius 5. $AB$ and $CD$ are both diameters of the larger circle. Let points $E$ and $F$ be the intersection points of $AB$ with the smaller circle, and $G$ be an intersection point of $CD$ with the smaller circle (see figure). Find the ratio of the areas of triangle $ABD$ and triangle $EFG$.




29. Eric wants to take a screenshot of how long he's been facetiming his friend, but being the mathematician he is, he refuses to take the picture unless the time is a palindrome and has at least one 5 in it. The app displays the time in the form of H:MM:SS; for example, the time of 3 hours 7 minutes and 21 seconds would be displayed as 3:07:21. Unfortunately, Eric just missed screenshotting at 3:45:43. For how many more seconds do the two friends have to talk until Eric can take a screenshot that satisfies his two requirements?

30. Let the roots of the polynomial $x^2 + 5x - 17$ be $p$ and $q$. What is the value of $\frac{p}{q} + \frac{q}{p}$?

31. Elaine finds herself at the origin and wants to travel to the point $(1, 5)$. On each turn, she can only go one unit up and one unit left OR one unit up and one unit right. If Elaine has an infinite number of turns, in how many ways can she accomplish her goal?

32. Shrek has a magic knife that can cut a slice of cheese in the ratio $1 : x$ for some fixed real $0 < x < 1$. Shrek uses the knife to cut a slice of cheese into two slices. Shrek uses the knife a second time to cut the bigger slice into two more slices. This procedure yields three pieces of cheese. It turns out that when the two smallest pieces are put together, they yield a slice of cheese which is identical to the largest piece. Find $x$.

33. Robot Greg has a robotic arm made up of 3 segments: one of length 4, one of length 2, and one of length 1. His robot arm is connected to his pet Lyp. One end of the segment of length 4 is connected to the point $(0, 0, 0)$ and its other end is connected to one end of the segment of length 2. The other end of the segment of length 2 is connected to one end of the segment of length 1, and the other end of the segment of length 1 is connected to his Lyp. At each connection between two segments, there is a freely-rotating joint that can make any bend in 3 dimensions. In addition, there is a freely-rotating joint at $(0, 0, 0)$. What is the volume of the set of all points the Lyp can visit in 3-D space?

34. Circle $\Gamma_1$ and circle $\Gamma_2$ intersect at points $X$ and $Y$. The radius of circle $\Gamma_1$ is 4 and the radius of circle $\Gamma_2$ is 4. If the tangent to circle $\Gamma_1$ at $X$ passes through the center of circle $\Gamma_2$, what is the distance between the centers of circles $\Gamma_1$ and $\Gamma_2$?

35. How many ways are there to rearrange the letters of "seventeen" so that the consonants are either in alphabetical order or in reverse alphabetical order?

36. Daniel draws the parabola $y = x^2 + 6x + 8$ on the blackboard. Then, Derek draws the parabola $y = -x^2 - x + 2$ on the blackboard. The two parabolas meet at the distinct points $A$ and $B$. Find the slope of the line $AB$.

37. Evaluate $\sqrt[2!]{2^{0!}}\,\sqrt[3!]{2^{1!}}\,\sqrt[4!]{2^{2!}}\,\sqrt[5!]{2^{3!}}\cdots$.

38. Let $P(x) = x^{2018} + x^{2017} + \cdots + x^2 + x + 1$ and $Q(x) = x^3 + 2x^2 + 3x + 2$. Let $M$ be the product of the roots of $P(Q(x))$. What is the least positive integer $y$ such that $2^y > M$?

39. Find the area of all possible points $P$ in a unit square such that for each side of the square, there exists a pair of points on it which form an equilateral triangle with point $P$.

40. Andrew the Sun Bear and Simon are 48 miles apart on an infinite flat plane. At the same time, they start running in a straight line in a random direction, with Andrew running at 20 miles per hour and Simon running at 12 miles per hour. The set of all possible points where Andrew and Simon could collide is a circle. If Simon had instead run at $x$ miles per hour, then the radius of this circle would be 13 miles smaller. Find the value of $x$.