Joe Holbrook Memorial Math Competition

4th Grade Solutions

October 13, 2019

1. The movie is a total of $2 \cdot 60 + 20 = 140$ minutes long, and Michael slept for 20 minutes. Thus, Michael slept for $\frac{20}{140} = \boxed{\frac{1}{7}}$ of the movie.

2. Using PEMDAS, $2 \times (0 - 1) + 9 = 2 \times (-1) + 9 = -2 + 9 = \boxed{7}$.

3. Recall that the formula for the area of a rectangle is length times width. As the area is 20 and the width is 5, the length is $20/5 = \boxed{4}$.

4. There are 7 days in a week. Thus the number of pounds of potatoes generated per day is $119/7 = \boxed{17}$.

5. We calculate Simon's height to be 67 inches and Andrew's height to be 76 inches. Subtracting, we find $76 - 67 = \boxed{9}$.

6. Simply perform division. We find $\boxed{5.75}$.

7. After simplification the equation becomes $12x = 24$. Solving for x, we find $x = \boxed{2}$.

8. Note that each corresponding pair of digits sum to 10. As there are 8 digits after the decimal point, the answer is $\boxed{11.1111111}$ (with 7 ones after the decimal point).

9. Since 2017, 2018, and 2019 form an arithmetic progression, $2017 + 2018 + 2019 = 3 \cdot 2018$. Hence the answer is $\frac{3 \cdot 2018}{6} = \frac{2018}{2} = \boxed{1009}$.

10. The prime factorization of 2019 is $3 \cdot 673$. Thus our answer is $3 + 673 = \boxed{676}$.

11. Note that 5.1 hours equals $5.1 \times 60 = 306$ minutes. Since Charles recited 100 words per minute, in the 5.1 hours, Charles recited $306 \times 100 = \boxed{30600}$ words, which is our answer.

12. As $2 \cdot 5 = 10$ divides $9!$, the last digit is $\boxed{0}$.

13. It is given that 3 and 5 are both less than 4. Further, $4 < 1 < 2$. Thus, $\boxed{4}$ is the median.

14. Let x be the answer to this question. Then $x = 1009 + \frac{x}{2019}$, implying $x = \boxed{1009.5}$.

15. Let the three integers be $n - 1, n, n + 1$. Then their sum is $(n - 1) + n + (n + 1) = 3n = 15$, and hence the middle integer is $\frac{15}{3} = 5$. Since the three integers are 4, 5, and 6, their product is $4 \cdot 5 \cdot 6 = \boxed{120}$.

16. When a number x is entered into the machine, the machine adds 6, which gives $x + 6$. Next, it multiplies by 2 and subtracts 4, which yields $2(x+6)-4$. We are given that $2(x+6)-4 = 14$. Solving this equation yields $x = \boxed{3}$.

17. Note that $\langle 6 \rangle = 1+2+3 = 6$, hence $\langle\langle 6 \rangle\rangle = \langle 6 \rangle = 6$, and $\langle\langle\langle 6 \rangle\rangle\rangle = \langle 6 \rangle = \boxed{6}$. (Fun fact: in number theory, we call n a *perfect number* if $\langle n \rangle = n$.)

18. Let b be the number of big apples and let s be the number of small apples. We are given that $b + s = 22$ and $5b + 3s = 94$. Solving the system of equations, we find $s = \boxed{8}$.

19. Using conversion factors, we find the answer of $100 \text{ flips} \cdot \frac{3 \text{ flaps}}{2 \text{ flips}} \cdot \frac{6 \text{ flops}}{5 \text{ flaps}} = \boxed{180}$ flops.

20. We use PEMDAS. We have $3 \circ 2 = 3^2 - 2^2 = 9 - 4 = 5$. This leaves us with $7 \circ 5$, or $7^2 - 5^2 = 49 - 25 = \boxed{24}$.

21. Jerry quacks at times that are multiples of 10 minutes after 12:00, and Akash quacks at times that are multiples of 18 minutes after 12:00. They will quack at the same time at multiples of both 10 and 18 minutes after 12:00. The least common multiple of 10 and 18 is 90. Thus, they will both quack 90 minutes after 12:00, at $\boxed{\text{1:30 PM}}$.

22. To calculate the exponent tower, we first calculate $1^9 = 1$, then $0^{1^9} = 0^1 = 0$, and finally $2^{0^{1^9}} = 2^0 = 1$. Hence, the original expression equals $1 \times 2018 - 2017 = 2018 - 2017 = \boxed{1}$, our answer.

23. Let x be the number of people who voted twice. Let y be the number of people who voted once. We get the system of equations $x + y = 2020$ and $2x + y = 2019 + 2018$. Solving this yields $x = \boxed{2017}$.

24. Since x must be real, $k \geq 0$. Since x must be an integer, k must be a square. There are $\boxed{10}$ squares from 0 to 99 inclusive.

25. Isolating the sum $a^4 + b^4$, we find $a^4 + b^4 = 97$. Clearly, a and b must be less than 4. Testing $a = 3$, we find $b^4 = 97 - 3^4 = 16 \implies b = 2$. By symmetry the pair $(a, b) = (2, 3)$ satisfies the equation. Either way, the answer is $a + b = \boxed{5}$.

26. If Charles was not carrying any cookies, he would be able to walk to the cookie fairy in $\frac{48}{6} = 8$ hours. However, since he is carrying 120 pounds of cookies, he can only walk at $6 - \frac{120}{30} = 2$ miles per hour, so it will take him $\frac{48}{2} = 24$ hours to walk to the cookie fairy. This takes him an extra $24 - 8 = \boxed{16}$ hours.

27. *Method 1:* The sum of the coefficients of a polynomial $P(x)$ is simply $P(1)$. To see this, note that if $P(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$, then

$$P(1) = a_n 1^n + a_{n-1} 1^{n-1} + \ldots + a_1 1 + a_0$$
$$= a_n + a_{n-1} + \ldots + a_1 + a_0,$$

precisely the sum of the coefficients of $P(x)$. Hence, the answer is $(2 \cdot 1 + 1)^3 = 3^3 = \boxed{27}$.

Method 2: Expanding the given polynomial, we find

$$(2x+1)^3 = (2x+1)^2(2x+1) = (4x^2 + 4x + 1)(2x + 1)$$
$$= 8x^3 + 12x^2 + 6x + 1.$$

Thus the sum of the coefficients is $8 + 12 + 6 + 1 = 27$, as above.

28. Derek reaches the shore in $\dfrac{600}{75} = 8$ minutes, after which there are 2 people in the water. Sameer reaches the shore in $\dfrac{600}{60} = 10$ minutes, after which only Daniel is in the water. Thus, exactly 2 people are in the water for $10 - 8 = \boxed{2}$ minutes.

29. The product is divisible by $2 \cdot 5 = 10$, so the remainder is either 0 or 10. The remainder can't be 0, as the product is only divisible by one power of 2. Hence our answer is $\boxed{10}$.

30. Let c denote the number of chickens on the farm, and let p denote the number of pigs on the farm. Since there are 21 heads total, including Eric's, $c + p + 1 = 21$. Also, since there are 52 feet total, including both of Eric's, $2c + 4p + 2 = 52$. Solving this system, we find the answer of $c = \boxed{15}$ chickens.

31. The first two sentences are enough to solve the problem. The difference between two prime numbers can only be odd if the smaller number is 2. Thus Eric was $\boxed{2}$ years old and Charles was 17.

32. Note that a can be negative. Hence, we take a to be the most negative it can be, so long b remains even. The least possible value of b is 2, which corresponds to the most negative value of $a = -128$, yielding a sum of $-128 + 2 = \boxed{-126}$.

33. The maximum possible area of a convex polygon with a fixed perimeter occurs when the polygon is equilateral. Thus, the maximum area of $MARVINYU$ occurs when the octagon is equilateral with side length $24/8 = 3$. To quickly compute the area of $MARVINYU$, denote $P_1 = MA \cap RV$, $P_2 = RV \cap IN$, $P_3 = IN \cap YU$, and $P_4 = YU \cap MA$. Then $P_1P_2P_3P_4$ is a square with side length $3 + 3\sqrt{2}$. The area of octagon $MARVINYU$ is equal to the area of $P_1P_2P_3P_4$ minus four times the area of the right isosceles triangle P_1RA. Thus, our answer is

$$(3 + 3\sqrt{2})^2 - 4 \cdot \frac{1}{2} \cdot \left(\frac{3\sqrt{2}}{2}\right)^2 = \boxed{18 + 18\sqrt{2}}.$$

34. Recall that if a number is divisible by 9, the sum of digits of the number must be divisible by 9. As 9 divides x, 9 must divide $2+5+9+1+A+0+B = 17+A+B$, implying that $A+B$ leaves a remainder of 1 when divided by 9. Similarly, as 9 divides y, 9 must divide $1+0+2+4+2+A+B+C = 9+A+B+C$, implying that $A+B+C$ leaves a remainder of 0 when divided by 9. It follows that C must leave a remainder of 8 when divided by 9. As C is a digit between 0 and 9, C must equal $\boxed{8}$, our answer.

35. Note that $|x-y| = \sqrt{(x-y)^2}$. Hence, to find $|x-y|$, it suffices to compute $(x-y)^2$. To finish, note that

$$\begin{aligned}(x-y)^2 &= x^2 - 2xy + y^2 \\ &= x^2 + 2xy + y^2 - 4xy \\ &= (x+y)^2 - 4xy \\ &= 37 - 3 \cdot 4 \\ &= 25,\end{aligned}$$

thus $|x-y| = \boxed{5}$.

36. The hamster's volume during summer is $\pi r^2 \cdot 2r = 2\pi r^3$. The hamster's volume during winter is $\pi(r+1)^2 \cdot (2r-1)$. As the hamster's volume is conserved, we have

$$2\pi r^3 = \pi(r+1)^2 \cdot (2r-1) \implies 2r^3 = (r+1)^2(2r-1)$$

$$\implies 2r^3 = 2r^3 + 3r^2 - 1 \implies 3r^2 = 1.$$

Solving, we find $r = \boxed{\dfrac{\sqrt{3}}{3}}$, as desired.

37. Since there are four seats around the table, Simon must sit across from Doug. Since Doug must sit next to Sumner, Sumner must either sit to the right or left of Doug. If Sumner sits to the right of Doug, then Andy must sit to the left of Doug, and similarly, if Sumner sits to the left of Doug, then Andy must sit to the right of Doug. These two cases are rotationally distinct, and hence there are $\boxed{2}$ ways to seat the four people.

38. Let the isosceles triangle have sides a, a and b. As the perimeter is 60, $2a+b = 60$. Further, the triangle inequality implies $2a > b$. Hence, $60 = 2a + b > 2b \implies b < 30$. As $b = 60 - 2a$ must be even, b can be any even integer between 2 and 28, inclusive. Each of these 14 values of b yields a non-degenerate isosceles triangle, and hence, our answer is $\boxed{14}$.

39. We have $A = 3 + \dfrac{1}{A}$, hence $A^2 - 3A - 1 = 0$. Solving gives us $A = \dfrac{3 \pm \sqrt{13}}{2}$. Since $\sqrt{13} > 3$, we must choose the positive solution, implying $A = \dfrac{3+\sqrt{13}}{2}$. Similarly, $B = \sqrt{3+B}$, hence $B^2 - B - 3 = 0$ and $B = \dfrac{1+\sqrt{13}}{2}$. Thus, $A - B = \boxed{1}$.

40. By the Pythagorean Theorem on $\triangle ABC$, $AC = \sqrt{3^2+4^2} = 5$. By the Pythagorean Theorem on $\triangle CDA$, $DA = \sqrt{5^2 - 1^2} = 2\sqrt{6}$. Since $\triangle ABC$ and $\triangle CDA$ are right triangles, their areas are $\dfrac{AB \cdot BC}{2} = 6$ and $\dfrac{CD \cdot DA}{2} = \sqrt{6}$, respectively. Since the area of $ABCD$ equals the sum of the areas of $\triangle ABC$ and $\triangle CDA$, our answer is $\boxed{6+\sqrt{6}}$.