Joe Holbrook Memorial Math Competition

5th Grade

October 13, 2019

General Rules

- You will have **75 minutes** to solve **40 questions**. Your score is the number of correct answers.
- Only answers recorded on the answer sheet will be graded.
- This is an individual test. Anyone caught communicating with another student will be removed from the exam.
- Scores will be posted on the website. Please do not forget your ID number, as that will be the sole means of identification for the scores.
- You may not use the following aids:
  - Calculator or other computing device
  - Compass
  - Protractor
  - Ruler or straightedge

  In addition, you must use the scrap paper supplied by the proctors.

Other Notes

- Write legibly. If the graders cannot read your answer, you will be given no credit for that question.
- Fractions should be written in lowest terms. Please convert all mixed numbers into improper fractions.
- For constants such as e or π, do not approximate your answer: for example, if the answer to a question is 7π, then you should not write 22 or 21.99.
- You do not need to write units in your answers.
- Rationalize all denominators. In addition, numbers within a square root must be squarefree, e.g. $\sqrt{63}$ should be written as $3\sqrt{7}$.
- Ties will be broken by the number of correct responses to questions 31 through 40. Further ties will be broken by the number of correct responses in the last five questions.

1. What is the value of $2 \times (0 - 1) + 9$?
2. What is the length of a rectangle with width 5 and area 20?
3. Susan has a potato farm that generates an average of 119 pounds of potatoes per week. On average, how many pounds of potatoes are generated per day?
4. Simon is 5 feet 7 inches tall, while his brother Andrew is 6 feet 4 inches. What is the positive difference between their heights, in inches?
5. Express $\frac{23}{4}$ in decimal form.
6. If $2 \cdot x + 0 \cdot x + 1 \cdot x + 9 \cdot x = 24$, what is x?
7. Compute $1.23456789 + 9.87654321$.
8. Compute $\frac{2017 + 2018 + 2019}{6}$.
9. For his final English project, Charles recited the play *Hamlet* at 100 words per minute. Given that it took Charles 5.1 hours to recite the play, how many words are there in *Hamlet*?
10. Let $n!$ denote the product of all positive integers less than or equal to n. Find the units digit of $9!$.
11. In the land of BCAmerica, numbers work differently. For the set of five numbers {1,2,3,4,5}, the following inequalities are true: $3 < 4$, $1 < 2$, $4 < 1$, and $5 < 4$. Which number in the set is the median?
12. Expressed as a decimal, what is 1009 added to $\frac{1}{2019}$ of the answer to this question?
13. The sum of 3 consecutive integers is 15. What is the product of the three integers?
14. When given a number, a machine adds 6 to the number, multiplies the result by 2, subtracts 4, and outputs the result. A number is entered into the machine, and the machine outputs 14. What number was entered into the machine?
15. Let $\langle n \rangle$ denote the sum of all positive divisors of n, excluding n itself. For example, $\langle 4 \rangle = 1 + 2 = 3$. What is $\langle\langle\langle 6 \rangle\rangle\rangle$?
16. If 2 flips equal 3 flaps, and 5 flaps equal 6 flops, how many flops equal 100 flips?
17. Define the function $a \circ b$ to equal the quantity $a^2 - b^2$. Compute $7 \circ (3 \circ 2)$.
18. Jerry and Akash sit in a field and quack occasionally. Jerry quacks every 10 minutes, and Akash quacks every 18 minutes. They both quack at 12:00 PM. When is the next time that they quack simutaneously?
19. Calculate $2^{0^{1^9}} \times 2018 - 2017$.
20. Abhinav is running for president against Susan. Everyone voted for either Abhinav, Susan, or both. Abhinav received 2019 votes, Susan received 2018 votes, and a total of 2020 students voted. How many people voted for both candidates?
21. For how many integers $k < 100$ does the equation $k = x^2$ have an integer solution x?
22. Let a and b be positive integers such that $1^4 + 5^4 + 6^4 + a^4 + b^4 = 2019$. Find $a + b$.
23. If $2^x = 25$, what is $2^{\frac{x}{2}+3}$?
24. Find the sum of the coefficients of the polynomial $(2x + 1)^3$.
25. What is the remainder when the product of the primes less than 100 is divided by 20?
26. Eric the human, his chickens, and his pigs live on the Ming Farm. Given that there are 21 heads and 52 feet on the farm, how many chickens are on the farm?
27. One day, Charles and his younger brother Eric realized that their ages were both prime numbers. Charles noted that the difference between their ages was 15. Eric then summed the cubes of their ages and got 4921. What was Eric's age?
28. If a and b are integers such that $a^b = 2^{14}$, what is the minimum possible value of $a + b$?

29. There exist three digits A, B, and C such that both the numbers $x = 2591A0B$ and $y = 10242ABC$ are multiples of 9. What is C? (Here, A, B, and C represent digits in the decimal representations of x and y.)

30. If $(x+y)^2 = 37$ and $xy = 3$, what is $|x-y|$?

31. Barry's College of Academics has three departments: Science, Arts, and Humanities. Each department has a positive number of students, no two departments have the same number of students, and no student is a member of more than one department. On Field Day, $\frac{1}{4}$ of Science students, $\frac{1}{3}$ of Arts students, and $\frac{1}{2}$ of Humanities students wore blue. If exactly $\frac{1}{3}$ of students at Barry's College of Academics wore blue on Field Day, what is the smallest possible number of students who attend Barry's College of Academics?

32. Simon and Doug are enemies and refuse to sit together. On the other hand, Doug and Sumner are best friends and insist on sitting next to each other. How many distinct ways are there to seat Simon, Doug, Sumner, and Andy around a circular table with four seats? Two arrangements are considered indistinguishable if one can be obtained from the other by a rotation.

33. JazzyZ graphs the parabolas $y = x^2$ and $y = -x^2 + 4$ on the coordinate plane. Let V_1 and V_2 be the vertices of the former and latter parabolas, respectively, and let A and B be the intersection points of the two parabolas. Find the perimeter of quadrilateral AV_1BV_2.

34. How many non-degenerate isosceles triangles with integer side lengths have perimeter 60? (An equilateral triangle is considered to be isosceles.)

35. Let $A = 3 + \dfrac{1}{3 + \frac{1}{3+\frac{1}{3+\ldots}}}$ and $B = \sqrt{3+\sqrt{3+\sqrt{3+....}}}$. Evaluate $A - B$.

36. Find the real value of x which satisfies

$$20002x^3 + 10001x + 90009 = 20192019.$$

37. In quadrilateral $ABCD$, $\angle ABC = \angle CDA = 90^\circ$. Furthermore, $AB = 3, BC = 4$, and $CD = 1$. What is the area of quadrilateral $ABCD$?

38. Define a positive integer to be *Susanian* if it satisfies the following two properties: the sum of the digits is 9 and the number is divisible by 15. How many Susanian integers are there less than 500?

39. Compute the sum

$$\left\lfloor \frac{2^0}{3} \right\rfloor + \left\lfloor \frac{2^1}{3} \right\rfloor + \left\lfloor \frac{2^2}{3} \right\rfloor + \ldots + \left\lfloor \frac{2^{11}}{3} \right\rfloor,$$

where $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x.

40. Let $f(x)$ equal the area of the parallelogram with vertices $(0,0), (x,1), (1,x)$, and $(x+1, x+1)$. Compute the infinite sum $\dfrac{1}{f(2)} + \dfrac{1}{f(3)} + \dfrac{1}{f(4)} + \ldots$.