Joe Holbrook Memorial Math Competition

6th Grade

October 13, 2019

General Rules

- You will have **75 minutes** to solve **40 questions**. Your score is the number of correct answers.
- Only answers recorded on the answer sheet will be graded.
- This is an individual test. Anyone caught communicating with another student will be removed from the exam.
- Scores will be posted on the website. Please do not forget your ID number, as that will be the sole means of identification for the scores.
- You may not use the following aids:
  - Calculator or other computing device
  - Compass
  - Protractor
  - Ruler or straightedge

  In addition, you must use the scrap paper supplied by the proctors.

Other Notes

- Write legibly. If the graders cannot read your answer, you will be given no credit for that question.
- Fractions should be written in lowest terms. Please convert all mixed numbers into improper fractions.
- For constants such as e or π, do not approximate your answer: for example, if the answer to a question is 7π, then you should not write 22 or 21.99.
- You do not need to write units in your answers.
- Rationalize all denominators. In addition, numbers within a square root must be squarefree, e.g. $\sqrt{63}$ should be written as $3\sqrt{7}$.
- Ties will be broken by the number of correct responses to questions 31 through 40. Further ties will be broken by the number of correct responses in the last five questions.

1. What is the value of $2 \times (0 - 1) + 9$?
2. What is the length of a rectangle with width 5 and area 20?
3. Simon is 5 feet 7 inches tall, while his brother Andrew is 6 feet 4 inches. What is the positive difference between their heights, in inches?
4. Express $\frac{23}{4}$ in decimal form.
5. If $2 \cdot x + 0 \cdot x + 1 \cdot x + 9 \cdot x = 24$, what is x?
6. Compute $\frac{2017 + 2018 + 2019}{6}$.
7. For his final English project, Charles recited the play *Hamlet* at 100 words per minute. Given that it took Charles 5.1 hours to recite the play, how many words are there in *Hamlet*?
8. Let $n!$ denote the product of all positive integers less than or equal to n. Find the units digit of $9!$.
9. Expressed as a decimal, what is 1009 added to $\frac{1}{2019}$ of the answer to this question?
10. The sum of 3 consecutive integers is 15. What is the product of the three integers?
11. When given a number, a machine adds 6 to the number, multiplies the result by 2, subtracts 4, and outputs the result. A number is entered into the machine, and the machine outputs 14. What number was entered into the machine?
12. Let $\langle n \rangle$ denote the sum of all positive divisors of n, excluding n itself. For example, $\langle 4 \rangle = 1 + 2 = 3$. What is $\langle\langle\langle 6 \rangle\rangle\rangle$?
13. A big apple costs \$5 and a little apple costs \$3. Favid spent \$94 buying 22 apples. How many small apples did Favid buy?
14. If 2 flips equal 3 flaps, and 5 flaps equal 6 flops, how many flops equal 100 flips?
15. Jerry and Akash sit in a field and quack occasionally. Jerry quacks every 10 minutes, and Akash quacks every 18 minutes. They both quack at 12:00 PM. When is the next time that they quack simutaneously?
16. Abhinav is running for president against Susan. Everyone voted for either Abhinav, Susan, or both. Abhinav received 2019 votes, Susan received 2018 votes, and a total of 2020 students voted. How many people voted for both candidates?
17. For how many integers $k < 100$ does the equation $k = x^2$ have an integer solution x?
18. Let a and b be positive integers such that $1^4 + 5^4 + 6^4 + a^4 + b^4 = 2019$. Find $a + b$.
19. Vfire must walk 48 miles to reach the cookie fairy. He walks at 6 miles per hour when he is not carrying anything. However, he is forced to carry 120 pounds of cookies, and every 30 pounds of cookies makes him 1 mile per hour slower. How many fewer hours would it take Vfire to walk to the cookie fairy if he was not carrying anything?
20. If $2^x = 25$, what is $2^{\frac{x}{2}+3}$?
21. Find the sum of the coefficients of the polynomial $(2x + 1)^3$.
22. Daniel, Derek, and Sameer are canoeing during a thunderstorm, and their canoe capsizes 600 feet from the shore. Daniel drifts towards the shore at a rate of 20 feet per minute, Sameer swims at a rate of 60 feet per minute, and Derek swims at a rate of 75 feet per minute. For how many minutes are there exactly 2 people in the water?
23. Eric the human, his chickens, and his pigs live on the Ming Farm. Given that there are 21 heads and 52 feet on the farm, how many chickens are on the farm?
24. One day, Charles and his younger brother Eric realized that their ages were both prime numbers. Charles noted that the difference between their ages was 15. Eric then summed the cubes of their ages and got 4921. What was Eric's age?
25. If a and b are integers such that $a^b = 2^{14}$, what is the minimum possible value of $a + b$?

26. Points M, A, R, V, I, N, Y, and U lie in the plane such that

$$MA + AR + RV + VI + IN + NY + YU + UM = 24.$$

Given that $MARVINYU$ is a convex polygon with vertices in that order, find the maximum possible area of polygon $MARVINYU$.

27. There exist three digits A, B, and C such that both the numbers $x = 2591A0B$ and $y = 10242ABC$ are multiples of 9. What is C? (Here, A, B, and C represent digits in the decimal representations of x and y.)

28. If $(x+y)^2 = 37$ and $xy = 3$, what is $|x-y|$?

29. Simon and Doug are enemies and refuse to sit together. On the other hand, Doug and Sumner are best friends and insist on sitting next to each other. How many distinct ways are there to seat Simon, Doug, Sumner, and Andy around a circular table with four seats? Two arrangements are considered indistinguishable if one can be obtained from the other by a rotation.

30. JazzyZ graphs the parabolas $y = x^2$ and $y = -x^2 + 4$ on the coordinate plane. Let V_1 and V_2 be the vertices of the former and latter parabolas, respectively, and let A and B be the intersection points of the two parabolas. Find the perimeter of quadrilateral AV_1BV_2.

31. How many non-degenerate isosceles triangles with integer side lengths have perimeter 60? (An equilateral triangle is considered to be isosceles.)

32. Let $A = 3 + \cfrac{1}{3 + \frac{1}{3 + \frac{1}{3 + \dots}}}$ and $B = \sqrt{3 + \sqrt{3 + \sqrt{3 + \dots}}}$. Evaluate $A - B$.

33. Find the real value of x which satisfies

$$20002x^3 + 10001x + 90009 = 20192019.$$

34. In quadrilateral $ABCD$, $\angle ABC = \angle CDA = 90^\circ$. Furthermore, $AB = 3, BC = 4$, and $CD = 1$. What is the area of quadrilateral $ABCD$?

35. Define a positive integer to be *Susanian* if it satisfies the following two properties: the sum of the digits is 9 and the number is divisible by 15. How many Susanian integers are there less than 500?

36. Compute the sum

$$\left\lfloor \frac{2^0}{3} \right\rfloor + \left\lfloor \frac{2^1}{3} \right\rfloor + \left\lfloor \frac{2^2}{3} \right\rfloor + \dots + \left\lfloor \frac{2^{11}}{3} \right\rfloor,$$

where $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x.

37. Let $f(x)$ equal the area of the parallelogram with vertices $(0,0), (x,1), (1,x)$, and $(x+1, x+1)$. Compute the infinite sum $\frac{1}{f(2)} + \frac{1}{f(3)} + \frac{1}{f(4)} + \dots$.

38. Let $A_0B_0C_0$ be a triangle with $A_0B_0 = 13$, $B_0C_0 = 14$, $C_0A_0 = 15$. For $n > 0$, define the triangles $A_nB_nC_n$ such that A_n is the midpoint of $\overline{A_{n-1}B_{n-1}}$, B_n is the midpoint of $\overline{B_{n-1}C_{n-1}}$, and C_n is the midpoint of $\overline{C_{n-1}A_{n-1}}$. Find $A_0B_0 + A_1B_1 + A_2B_2 + \cdots$.

39. Let $ABCD$ be an isosceles trapezoid with $AB \parallel CD$ with $AB = 5$, $CD = 15$, and $\angle ADC = 60^\circ$. If E is the intersection of $\overline{AC}$ and $\overline{BD}$ and O is the circumcenter of $\triangle ABC$, compute EO.

40. Simon the farmer has four trees, numbered 1, 2, 3, and 4. There are a total of 3 pieces of fruit hanging on the trees: 2 identical blue fruits and 1 green fruit. Two trees are said to *match* if the number of blue fruits on each are equal and if the number of green fruits on each are equal. Assume that each fruit is equally likely to be located on each of the trees. If Tree 2 doesn't match Tree 4, and if Tree 1 doesn't match Tree 3, what is the probability that Tree 3 matches Tree 2?