Joe Holbrook Memorial Math Competition

4th Grade Solutions

October 18, 2020

1. We perform the addition/subtraction of $76 + 12 - 2 = \boxed{86}$.

2. From the fact that 4 scores and 7 years is equivalent to 87 years, we can see that a score is 20 years. Therefore, 2020 years is equivalent to $\frac{2020}{20} = \boxed{101}$ scores.

3. There are 14 choices for a consonant and 10 choices for a vowel, so $14 * 10 = \boxed{140}$.

4. Alex will take $\frac{3000}{50} = 60$ minutes, and Yoland will take $\frac{3000}{120} = 25$ minutes, so $60 - 25 = \boxed{35}$ minutes.

5. $2.5 \text{ hr} * \frac{60 \text{ min}}{1 \text{ hr}} = 150 \text{ min}$; $\frac{5 \text{ things}}{1 \text{ min}} * 150 \text{ min} = \boxed{750}$

6. There are $1 - \frac{1}{2} = \frac{1}{2}$ of the original apples remaining after Pete. Of those, there are $1 - \frac{1}{3} = \frac{2}{3}$ of those remaining after Sam. Therefore, the total number of apples remaining after both is $12 * \frac{1}{2} * \frac{2}{3} = \boxed{4}$.

7. This problem may be solved as a system of equations. Let x denote the number of cows and let y denote the number of chicken. Then $x + y = 15$ and $4x + 2y = 38$. Solving this equation using substitution or elimination will yield that $x = 4$. Interestingly, this problem can also be solved using a "math team trick." Consider having all chicken, then there will be 30 legs. Converting a chicken to a cow will allow 2 more legs. Therefore, to increase the number of legs from 30 to 38, one should add $\frac{8}{2} = \boxed{4}$ cows.

8. The least common multiple of 10 $(2 \cdot 5)$, 15 $(3 \cdot 5)$, and 4 $(2 \cdot 2)$ is $2 \cdot 2 \cdot 3 \cdot 5 = \boxed{60}$

9. The number of four letter words is $500 - (27 + 166 + 92) = 215$, so the probability of one being selected is $\frac{215}{500} = \frac{43}{100}$. So $43 + 100 = \boxed{143}$.

10. There are 8 primes, namely 2, 3, 5, 7, 11, 13, 17, 19, between 1 and 20. Therefore, the probability is $\frac{8}{20} = \frac{2}{5}$ and $p + q = \boxed{7}$.

11. Let the number you gave Jennifer be x. Then she multiplies x by 6 and subtracts 10, returning the expression $6x - 10$. We know that the number she returned to you is the number that you gave her, so $6x - 10 = x$. We move all the x terms to the left side of the equation and the numbers to the right, giving us $5x = 10$. Dividing both sides by 5 gives $x = \boxed{2}$.

12. The probability that Erik doesn't damage his computer at a single error message is $1 - \frac{2}{3} = \frac{1}{3}$. This happens 6 times in a row with probability $\left(\frac{1}{3}\right)^6 = \frac{1}{729}$. $1 + 729 = \boxed{730}$

13. $\frac{9 \cdot 12}{2} = 54$ boxes can fit vertically, and $\frac{16 \cdot 12}{3} = 64$ boxes can fit horizontally. $54 \cdot 64 = \boxed{3456}$

14. Using brute force, we find that the sum of the first two terms is 225. The last term is $0 \pmod{1000}$, so the answer is $\boxed{225}$

15. We should use a common denominator to make these fractions easier to understand. This gives us
$$\frac{5}{45} < \frac{x}{45} < \frac{18}{45}.$$
Multiplying this inequality by 45 gives us
$$5 < x < 18.$$
Thus x has $18 - 5 - 1 = \boxed{12}$ possible values of x.

16. We know that 2L=2000 mL. Divide 2000 by 1+2+3+4 to get the smallest cup's volume, which is 200 mL. Using this fact, the cups will have volumes of 200, 400, 600, 800 mL each. The answer is $\boxed{600}$ mL.

17. $\frac{0.4 \text{ mi}}{2 \text{ mph}} = 0.2$ hr of walking for Yul; $\frac{0.3 \text{ mi}}{5 \text{ mph}} = 0.06$ hr of jogging for Alicia. $0.2 - 0.06 = 0.14 \text{ hr} * \frac{60 \text{ min}}{1 \text{ hr}} * \frac{60 \text{ s}}{1 \text{ min}} = \boxed{504}$ seconds.

18. Just check every number: one, three, and seven all give $\boxed{2}$.

19. There are 27 unit cubes. The cubes are the center of each face are the only cubes with exactly one face painted. Therefore, there are 6 such cubes. This makes for the probability to be $6/27 = 2/9$. So, $\boxed{\text{a+b=11}}$.

20. We just need to subtract the volume of the flesh from the volume of the whole watermelon. The volume of a sphere is $\frac{4}{3}\pi r^3$. The radius of the watermelon is $\frac{12}{2} = 6$, while the radius of the flesh is $6 - 1 = 5$.

$$\frac{4}{3}\pi(6^3) - \frac{4}{3}\pi(5^3) = \frac{364\pi}{3}$$

$x = \boxed{367}$.

21. The probability that he only makes the football team is $\frac{1}{5} \cdot \frac{3}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} = \frac{6}{120}$. The probability that he only makes the basketball team is $\frac{4}{5} \cdot \frac{1}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} = \frac{8}{120}$. The probability that he only makes the tennis team is $\frac{4}{5} \cdot \frac{3}{4} \cdot \frac{1}{3} \cdot \frac{1}{2} = \frac{12}{120}$. The probability that he only makes the soccer team is $\frac{4}{5} \cdot \frac{3}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} = \frac{24}{120}$. This sums to $\frac{50}{120} = \frac{5}{12} \to 5 + 12 = \boxed{17}$.

22. "We just use dimensional analysis:

$$20 \text{ Susans} \cdot \frac{10 \text{ Suzans}}{1 \text{ Susan}} \cdot \frac{2 \text{ Suzettes}}{7 \text{ Suzans}} \cdot \frac{12 \text{ Suzys}}{2 \text{ Suzettes}} \cdot \frac{7 \text{ Susies}}{10 \text{ Suzys}} = \boxed{240} \text{ Susies}$$

23. We look to maximize the number at every stage of picking the digits. The largest 2-digit prime number is 97, and conveniently, the largest 2-digit prime with a tens unit of 7 is 79. Therefore, we should begin with a 9, then a 7, then a 9, and repeat in this matter. Therefore, our answer is $\boxed{97979}$.

24. We first consider the two O's as one letter. There are then $\frac{5!}{2!} = 60$ ways to rearrange the letters P, OO, T, A, and T. From this, we subtract the number of rearrangements where the two T's are next to each other: $60 - 4! = \boxed{36}$.

25. If we graph the square the and inequality, this clearly becomes an area problem. We see that the area in the square but above the line $y = -x + 3$ is a triangle with legs of length of 1 each, so it's area is $\frac{1}{2}$. So the area under the line but contained within the square has an area of $\frac{7}{2}$. This over the total area of the square, 4, gives us $\frac{7}{8}$, so the answer will be the product of the numerator and denominator, which is $\boxed{56}$

26. Simon is driving for 100 minutes, which is equal to $\frac{5}{3}$ hours. $\frac{5}{3} \cdot 50 = \frac{250}{3}$ miles travelled. Simon travels $\frac{40}{60} \cdot 40 = \frac{80}{3}$ miles at first. Therefore, he must drive $\frac{250}{3} - \frac{80}{3} = \frac{170}{3}$ miles in 60 minutes. Therefore, he must drive $\frac{170}{3}$ miles per hour for 60 minutes, so the answer is $170 + 3 = \boxed{173}$.

27. This problem asks to find the sum of $1/2^1 + 1/2^2 + 1/2^3 \cdots + 1/2^8$. Either using the formula for a geometric series or with pattern detecting, one can evaluate this to be $\frac{255}{256}$. We conclude that the $p + q$ is $\boxed{511}$.

28. EF is a rectangle, so $EF = GC$. Notice that GC is perpendicular to AB, so GC is the height of the triangle. So, $GC * AB = area * 2 = AC * BC$, which gives $GC = \frac{12}{5}$. So the answer is $12 + 5 = \boxed{17}$

29. In order to find the sum of the coefficients, we can plug in $a = 1$ and $b = 1$. So, in essence, we are solving for n when $3^n = 729$. Intuitively, $\boxed{n = 6}$.

30. One "cycle" of Cel's is three songs and an ad, which takes $3 \cdot 4 + 1 = 13$ minutes, and similarly one "cycle" of Aakriti's takes $10 \cdot 4 + 1 = 41$ minutes. The least common multiple of 13 and 41 is $13 \cdot 41 = 533$ minutes, so every 533 minutes, there will be 1 minute of shared ad-listening. There are $5 \cdot 24 \cdot 60 = 7200$ minutes in five days, so $\left\lfloor \frac{7200}{533} \right\rfloor = \boxed{13}$.

31. Listing out the fractions or noticing the telescope as $\frac{(n+2)(n+4)}{(n+1)(n+3)}$, then the product becomes $\frac{12 \cdot 14}{2 \cdot 4} = \boxed{21}$

32. We start with the second person and notice that he used the digit 3. This means that this person must be speaking in base five. Therefore, we know that $20_5 = 10$ people speak base $3_5 = 3$ and $11_5 = 6$ people speak both. To determine what base the first person speaks in we notice that we must have $6 = 20_?$ where $? = 3$ or 5. We see that ? must be 3. Knowing that the first person speaks base 3, we have that $101_3 = 10$ people speak base $12_3 = 5$. Hence, by PIE, there are $10 + 10 - 6 = \boxed{14}$ residents.

33. We may quickly use stars and bars to find that the number of ways of splitting the indistinguishable balls is $\binom{5}{2} = 10$, and the number of ways of dividing the distinguishable balls is $3^3 = 27$. The product of these two is $\boxed{270}$.

34. We must have that $x^4 + 528 = k^2$. So, we can subtract x^4 from both sides and factor with difference of squares: $528 = (k + x^2)(k - x^2)$. Note that $k + x^2$ and $k - x^2$ must have the same parity. Since 528 $= 2^4 \cdot 3 \cdot 11$, the possibilities for $k + x^2$ and $k - x^2$ are $(24, 22), (44, 12), (132, 4)$. Both possibilities work $(x = 1, 4, 8)$, so the answer is $23^2 + 28^2 + 64^2 = \boxed{5937}$.

35. First we only consider the genre of each movie. There are then 10 orderings that satisfy Cole's condition: 5 with action being among the first three (specifically ARHRHR, RARHRH, RAHRHR, HRARHR, RHARHR) and 5 with action being among the last three. We multiply 10 by $3! \cdot 2! \cdot 1!$ since the movies are distinguishable, yielding $\boxed{120}$.

36. Consider the angle between the hour hand and the minute hand. Every minute, the minute hand moves forward $\frac{1}{60} \cdot 360 = 6$ degrees, and the hour hand moves forward $\frac{1}{60} \cdot 30 = \frac{1}{2}$ degrees. In other words, every minute the angle measure between the hands shrinks by $6 - \frac{1}{2} = \boxed{\frac{11}{2}}$ degrees.

    When the mantis is $5\sqrt{3}$ cm away from the fly, the hands and the line connecting the mantis and fly form a 30-60-90 triangle, and the angle between the hands is 60 degrees. Since it is 3 o'clock right now, the current angle measure is 90 degrees. If m represents the number of minutes before the angle measure is 60 degrees, then

$$60 = 90 - \frac{11}{2}m$$

$$m = \frac{60}{11}$$

    Rounded to the nearest minute, it will take $\boxed{5}$ minutes for the mantis to grab the fly.

37. Note that for each person, there is a $\frac{1}{100}$ chance they get assigned to themselves. Since there are 100 people, the expected value for the number of poeple that get assigned to themselves is $\frac{1}{100} \cdot 100 = \boxed{1}$.

38. Let the cube's vertices be denoted $ABCDEFGH$, where $ABCD$ and $EFGH$ are two squares that form two of the parallel faces of the cube. The four edges connecting these two squares are AE, BF, CG, and DH. Let A be the starting point, and G be the endpoint. First of all, note that in the no-fire case, the shortest path is of length $\sqrt{5}$, passing through A, the midpoint of BC and G. In fact, there are five other paths that work as well, similar to the first path, but instead passing through the midpoints of HD, EF, CD, HE, or BF. Note that each of these paths only passes through two of the faces of the cube, and for each path, these two faces are distinct. It follows that even if any two faces are set on fire, by the pigeonhole principle, at least one of the 6 paths will remain safe from fire, hence $\sqrt{5}$ is always possible, meaning the expected shortest path is $\sqrt{5}$. $8\sqrt{5} = \sqrt{320}$, which is really close to $\sqrt{324} = 18$, so our answer is $\boxed{18}$.

39. Any element of S must have the exponent of 11 in its prime factorization be greater than or equal to 1. For primes 2, 3, 5, and 7, they must be greater than or equal to zero. So, using the formula for the sum of an infinite geometric series, the answer is equal to $\left(1+\frac{1}{2^1}+\frac{1}{2^2}+\dots\right)\left(1+\frac{1}{3^1}+\frac{1}{3^2}+\dots\right)\cdot\left(1+\frac{1}{5^1}+\frac{1}{5^2}+\dots\right)\left(1+\frac{1}{7^1}+\frac{1}{7^2}+\dots\right)\left(1+\frac{1}{11^1}+\frac{1}{11^2}+\dots\right)=2\cdot\frac{3}{2}\cdot\frac{5}{4}\cdot\frac{7}{6}\cdot\frac{11}{10}=\frac{77}{16}$. The final answer is $\boxed{93}$.

40. We start by realizing that the number of coins in a pouch is equal to the sum of the factors of the bag's number. For instance, the sixth bag has 12 coins because People 1, 2, 3, and 6 dropped 1, 2, 3, and 6 coins, respectively. Therefore, we must determine which numbers from 1 through 100 have an odd sum of factors. The sum of factors for any number is calculated by taking each different prime factor and adding together all its powers up to the one that appears in the prime factorization, and then multiply all these sums together. For instance, Pouch 24 would have $(1+2+4+8)(1+3)=60$ coins. To generate an odd product, we must have only odd factors. Powers of 2 always generate odd factors since it is always $1+(powers\ of\ 2)$. Powers of other primes generate odd factors only when the power is even. For instance 3^2 generates $1+3+9=13$ but 3^3 generates $1+3+9+27=40$. With this in mind, all pouches with an odd number of coins will have powers of 2 and/or odd primes raised to even powers in their prime factorization. For powers of 2 we have $2, 2^2, 2^3, 2^4, 2^5, 2^6$, for powers of 3 we have $3^2, 3^4$, for powers of 5 we have 5^2, and for powers of 7 we have 7^2. Multiplying these numbers together yields 16 different numbers less than 100: 2, 4, 8, 16, 32, 64, 9, 18, 36, 72, 81, 25, 50, 100, 49, and 98. The final pouch is Pouch 1, yielding a total of $\boxed{17}$ pouches.