

# Joe Holbrook Memorial Math Competition

4th Grade

October 18, 2020

## General Rules

- You will have **75 minutes** to solve **40 questions**. Your score is the number of correct answers.
- Only answers recorded on the appropriate Google Form will be graded.
- You are to remain visible to your proctor at all times. Please have your video camera on during the exam.
- This is an individual test. Anyone caught communicating with another student or using technology in an inappropriate way will be removed from the exam.
- Scores will be posted on the website. Please do not forget your ID number, as that will be the sole means of identification for the scores.
- You may not use the following aids:
  - Calculator or other computing device
  - Compass
  - Protractor
  - Ruler or straightedge

## Other Notes

- All answers are integers. Please enter them with no spaces in between into the Google Form. For a negative integer, please enter  $-7$  not  $- 7$ .
- Do not include commas in your answers. For example, the number one thousand is to be entered 1000 not 1,000.
- You must not write units in your answers.
- Ties will be broken by the number of correct responses to questions 31 through 40. Further ties will be broken by the number of correct responses in the last five questions.

1. A guy who is 76 inches tall with his shoes on stands on a 12 inch tall bench, but takes off his 2-inch elevated shoes. How far off is the top of his head from the ground now?
2. If 4 scores and 7 years is equivalent to 87 years, how many scores is 2020 years?
3. Alicia is learning the Korean alphabet. If there are 14 basic consonants and 10 basic vowels, how many “basic letters” can she create? (A “basic letter” consists of one consonant followed by one vowel.)
4. Alex types at 50 words per minute (wpm) and Yolanda types at 120 wpm. If they both start typing a 3000 word essay, how many more minutes will it take Alex to finish?
5. Erez forgets things at a rate of 5 things per minute, and he doesn’t learn any new things. If his brain will be empty in exactly 2.5 hours, how many things are in his brain right now?
6. There are 12 apples in a basket. Pete eats  $\frac{1}{2}$  of the apples in the basket, then Sam eats  $\frac{1}{3}$  of the remaining apples. How many apples are in the basket after both Pete and Sam have eaten apples?
7. Agriculturalist Andy has a farm that consists of cows and chicken. Given that there are 15 heads and 38 legs in Andy’s farm, how many cows are there?
8. In Noah’s backyard, a bird chirps once every 10 seconds, a frog croaks once every 15 seconds, and his dog barks once every 4 seconds. If the three animals just made sounds simultaneously, how many seconds later will they again make a sound in unison?
9. In a very small dictionary, there are 500 words and they all consist of either 3, 4, 5, or 6 letters. There are 27 three letter words, 166 five letter words, and 92 six letter words. If a word is selected at random, and the probability it has four letters can be expressed as  $\frac{a}{b}$  in lowest terms, what is  $a + b$ ?
10. If the probability of choosing a prime number between 1 and 20, inclusive, is  $\frac{p}{q}$  in its most reduced form, what is  $p + q$ ?
11. When you give Jennifer a number, she multiplies the number by 6 and subtracts 10. One day you give her your favorite number and she gives you back the exact same number. What is the number you gave her?
12. Erik’s code doesn’t work, and every time an error message pops up, there is a  $\frac{2}{3}$  probability that he throws his computer out the window. If the probability that he makes it through 6 error messages without damaging his computer can be expressed as  $\frac{a}{b}$  in lowest terms, what is  $a + b$ ?
13. When using Zoom, each participant’s face appears in a 2 inch by 3 inch box. You have a projector screen that is 9 feet by 16 feet. Making sure that everyone’s box is fully inside the screen, what is the greatest number of participants you can see at once without overlapping?
14. Compute the remainder of  $\frac{(1 + 2)^2 + (1 + 2 + 3)^3 + (1 + 2 + 3 + 4)^4}{1000}$ .
15. For how many integers  $x$  is  $\frac{1}{9} < \frac{x}{45} < \frac{2}{5}$ ?
16. Sally is pouring lemonade out of a 2L jug to completely fill 4 cups of different volume. The ratios of the volumes are 1:2:3:4. What is the volume of the second largest cup in mL? (Note: 1L = 1000 mL)
17. Yul the Cül and Alicia the Felicia like to hang out together. They usually agree to meet at a street corner 0.3 miles from Alicia’s house and 0.4 miles from Yul’s house. If Yul likes to walk at a steady rate of 2 miles per hour and Alicia likes to jog at a steady rate of 5 miles per hour, how long, in seconds, will Alicia have to wait for Yul to arrive (assuming they leave their houses at the same time)?
18. Let  $f(x)$  be the non-negative difference between the number of letters in the English name of a number, and the actual number. For example,  $f(8) = |5 - 8| = 3$  since there are 5 letters in “eight”. What is the most common value of  $f(x)$  over the integers from 1 to 10 inclusive?
19. Given a 3 by 3 by 3 cube, consider painting the surface of the cube red. Now, decompose the cube into unit cubes. Let the probability that a randomly selected unit cube has exactly one side painted red be  $\frac{a}{b}$  in lowest terms, i.e. where  $(a, b) = 1$ . What is  $a + b$ ?

20. Yul the Cül likes to be cool in the summer, so she eats a lot of watermelon. She has a perfectly spherical watermelon of diameter 12 in., which consists of a sphere of flesh surrounded by a rind that has a uniform thickness of 1 in. If the volume of the surrounding rind can be written in the form  $\frac{a\pi}{b}$  cm<sup>3</sup> for relatively prime  $a, b$ , find  $a + b$ .
21. Justin simultaneously tries out for multiple sports teams. He either makes or doesn't make a team. The probability he makes the football team is  $\frac{1}{5}$ , the basketball team  $\frac{1}{4}$ , the tennis team  $\frac{1}{3}$ , and the soccer team  $\frac{1}{2}$ . If the probability that Justin the Jock makes exactly one team is  $\frac{a}{b}$ , where  $a, b$  are relatively prime, what is  $a + b$ ?
22. Simon can trade 7 Susies for 10 Suzys, 12 Suzys for 2 Suzettes, 7 Suzans for 2 Suzettes, and 10 Suzans for 1 Susan. How many Susies must he trade for 20 Susans?
23. Find the largest 5-digit number such that all the two digit numbers formed by pairing adjacent digits in the 5-digit number will be prime. The primes created in this way need not be unique.
24. How many ways are there to rearrange the letters in the word POTATO such that the two O's are next to each other and the two T's are not?
25. A point  $(x, y)$  is randomly and uniformly chosen inside the square with vertices  $(0,0)$ ,  $(0,2)$ ,  $(2,2)$ , and  $(2,0)$ . Given the probability that  $x + y < 3$  is  $\frac{m}{n}$  with  $m, n$  relatively prime, what is  $mn$ ?
26. Simon drives at a speed of 40 miles per hour for 40 minutes. He should drive  $\frac{a}{b}$  (where  $a$  and  $b$  are relatively prime) miles per hour for 60 minutes so that his average speed overall is 50 miles per hour. Find  $a + b$ .
27. Galactic Greg is searching for the secret to astronomy! Once he finds the secret, he stops searching, and the probability that he finds this secret on any given day  $n$  is  $\frac{1}{2^n}$ . The probability that Greg will find the secret before day 9 is  $\frac{p}{q}$  where  $(p, q) = 1$ . What is the  $p + q$ ?
28.  $ABC$  is a  $3 - 4 - 5$  triangle with  $AC = 3$ ,  $BC = 4$ ,  $AB = 5$ . Points  $E$ ,  $F$ , and  $G$  are on  $\overline{AC}$ ,  $\overline{BC}$ , and  $\overline{AB}$  respectively such that  $EC/FC = 4/3$  and  $EGFC$  is a rectangle. The length of  $\overline{EF}$  is  $a/b$ , where  $a$  and  $b$  are relatively prime. What is  $a + b$ ?
29. When the coefficients of  $(a + b + 1)^n$  are summed, the result was calculated to be 729. What is  $n$ ?
30. Cel doesn't have Spotify Premium, so every three songs, an advertisement plays. Aakriti has a slightly better version of Spotify that plays an advertisement every ten songs. All songs are 4 minutes, and all advertisements are 1 minute. If they each start playing music for the first time right now and keep it on for the next five days, how many minutes total will there be where they are both listening to an advertisement at the same time?
31. Yul has a Netflix addiction. On the  $n$ -th day she watches  $\frac{n^2 + 6n + 8}{n^2 + 4n + 3}$  hours. If she watched  $x_n$  hours in the  $n$ -th day, what is  $x_1 \cdot x_2 \cdots x_9 \cdot x_{10}$  in hours?
32. In a little town, people speak in either base three or base five and some people are fluent in both bases. However, they may only use one base in each sentence. A reporter interviewed two residents to figure out how many people lived in the town. The first person said "101 people speak base 12 and 20 people speak both" and the second person said "20 people speak base 3 and 11 speak both bases". How many people live in this little town?
33. How many ways are there to put three distinguishable and three other indistinguishable balls into three distinguishable bins?
34. Compute the sum of possible values for the expression  $x^4 + 528$  if it must be a perfect square, and  $x$  is a positive integer.
35. Cole is deciding what order he should watch his movies during his Netflix marathon. He has 6 different movies picked out: 3 are romantic comedies, 2 are horror, and 1 is action. He refuses to watch movies in the same genre back-to-back. How many different orders can Cole watch his six movies?

36. A fly and a mantis are sitting on the ends of the minute and hour hands of a clock respectively. The hour hand is 5 cm long and the minute hand is 10 cm long. When the mantis is within  $5\sqrt{3}$  cm of the fly, it can reach out and grab it. If it is 3 o'clock right now, how long will it be, to the nearest minute, until the mantis can reach out and grab the fly?
37. It is graduation and to celebrate, all 100 people in the class have decided to partake in a gift exchange. In this gift exchange, everyone is assigned a random person (with equal chance) to give a gift to. However, there is also a chance they are assigned themselves. Everyone is assigned exactly one person, and no person can be assigned to two people. What is the expected number of people that are assigned to themselves?
38. A wild traus (a lake-inhabiting animal) sitting at a corner of a unit cube wants to make its way to the vertex furthest from it by only crawling on the surface of the cube. However, two of the six sides of the cube are chosen randomly and set on fire, restricting the traus from crawling on them. The expected value for the length of the shortest path the traus can take to reach the desired vertex is  $a$ , what is the nearest integer to  $8a$ ?
39. Let  $S$  be the set of all positive integers whose largest prime factor is 11. The sum of the reciprocals of all the elements in  $S$  is  $\frac{m}{n}$  for relatively prime positive integers  $m$  and  $n$ . Compute  $m + n$ .
40. There are 100 empty coin pouches numbered 1 through 100 and 100 people numbered 1 through 100. Person  $n$  goes to every  $n$ th bag and drops  $n$  coins into it. For instance, the Person 1 goes to every pouch and drops 1 coin, Person 2 goes to every other pouch and drops 2 coins, and Person 3 goes to every third pouch and drops 3 coins, and so on. After all 100 people have gone, how many bags have an odd number of coins in them?