Joe Holbrook Memorial Math Competition

6th Grade

October 18, 2020

General Rules

- You will have **75 minutes** to solve **40 questions**. Your score is the number of correct answers.
- Only answers recorded on the appropriate Google Form will be graded.
- You are to remain visible to your proctor at all times. Please have your video camera on during the exam.
- This is an individual test. Anyone caught communicating with another student or using technology in an inappropriate way will be removed from the exam.
- Scores will be posted on the website. Please do not forget your ID number, as that will be the sole means of identification for the scores.
- You may not use the following aids:
  - Calculator or other computing device
  - Compass
  - Protractor
  - Ruler or straightedge

Other Notes

- All answers are integers. Please enter them with no spaces in between into the Google Form. For a negative integer, please enter -7 not $- \ 7$.
- Do not include commas in your answers. For example, the number one thousand is to be entered 1000 not $1,000$.
- You must not write units in your answers.
- Ties will be broken by the number of correct responses to questions 31 through 40. Further ties will be broken by the number of correct responses in the last five questions.

1. Alicia is learning the Korean alphabet. If there are 14 basic consonants and 10 basic vowels, how many "basic letters" can she create? (A "basic letter" consists of one basic consonant followed by one basic vowel.)

2. Erez forgets things at a rate of 5 things per minute. If his brain will be empty in exactly 2.5 hours and he learns nothing in those 2.5 hours, how many things are in his brain right now?

3. Alex types at 50 words per minute (wpm) and Yoland types at 120 wpm. If they both start typing a 3000 word essay, how many minutes longer will Alex take to finish?

4. Agriculturalist Andy has a farm that consists of cows and chicken. Given that there are 15 heads and 38 legs in Andy's farm, how many cows are there?(A cow can be assumed to have 4 legs and 1 head, a chicken can be assumed to have 2 legs and 1 head).

5. In Noah's backyard, a bird chirps once every 10 seconds, a frog croaks once every 15 seconds, and his dog barks once every 4 seconds. If the three animals just made sounds simultaneously, how many seconds later will they again make a sound in unison?

6. In a very small dictionary, there are 500 words and they all consist of either 3, 4, 5, or 6 letters. There are 27 three letter words, 166 five letter words, and 92 six letter words. If a word is selected at random, and the probability it has four letters can be expressed as $\frac{a}{b}$ in lowest terms, what is $a + b$?

7. If a fair twenty-sided die(labeled 1-20) is rolled with a fair ten-sided die(labeled 1-10) and the probability that the sum of the rolled numbers is odd is $\frac{a}{b}$ in lowest terms, what is $a + b$?

8. A palindrome is a number that is the same when read forwards or backwards (12321 is one such number). How many 5-digit palindromes are there?

9. Yellow pigs are storming BCA! If Suzy can get rid of them in 6 hours and Douglas can get rid of them in 3 hours, how many minutes would it take Suzy and Douglas to get rid of all the yellow pigs working together?

10. Autumn has a lot pages of homework that she doesn't want to do. She throws a third of the pages out the window. She then gives a fourth of the number of pages she has left to her dog. Autumn next gives one fifth of her remaining pages to Will. She then uses the remaining 12 pages as confetti. How many pages of homework did Autumn have in the beginning?

11. Simon wants to put a wooden frame around his poster of dimensions 10 inches by 20 inches. He wants the frame to be a 3-inch border on all sides of the poster. How much wood, in square inches, will Simon need to build his frame?

12. When using Zoom, each participant's face appears in a 2 inch by 3 inch box. If you have a projector screen that is 9 feet by 16 feet, what is the greatest number of participants you can see at once without overlapping?(Everyone's box must be fully inside the screen)

13. Currently, Prince George is 4 ft and Queen Elizabeth is 5 ft 4 inches. However, Prince George grows taller by 2 inches every year , while Queen Elizabeth shrinks by 0.2 inches every year. How many (whole) years must pass before Prince George is taller than Queen Elizabeth?

14. Yul the Cül likes to be cool in the summer, so she eats a lot of watermelon. She has a perfectly spherical watermelon of diameter 12 in., which consists of a sphere of flesh surrounded by a rind that has a uniform thickness of 1 in. If the volume of the surrounding rind can be written in the form $\frac{a\pi}{b}$ cm^3 for relatively prime a, b, find $a + b$.

15. A cylinder and cone have the same radius. The cone has half the height of the cylinder. The ratio of the volume of the cone to the volume of the cylinder is $\frac{a}{b}$, where a and b are relatively prime. Find $a + b$.

16. What is the sum of all numbers in the set $\{7, 17, 27, 37, \cdots, 707\}$ that can be written as the sum of 2 prime numbers?

17. What is the ones digit of $10^{300} + 7^{78} + 3^{33}$?

18. Find the largest integer n such that n is divisible by 5, the sum of the digits of n is divisible by 6, and $n < 1000$.

19. A point (x, y) is randomly and uniformly chosen inside the square with vertices (0,0), (0,2), (2,2), and (2,0). Given the probability that $x + y < 3$ is $\frac{m}{n}$ with m, n relatively prime, what is mn?

20. How many ordered pairs of positive integers satisfy that $2x + 5y = 100$?

21. M, A, T, and H are four different positive real numbers. $MA = 5, AT = 6, TH = 7$, and $HM = 8$. Let $M \cdot A \cdot T \cdot H = a\sqrt{b}$, where b has no square factors. What is $a + b$?

22. Dan is juggling a basketball, a tennis ball, a soccer ball, a ping-pong ball, and a bowling ball. He drops at least one item, but not all of them. How many possible sequences of the balls hitting the floor are there? For example, some possible sequences are a tennis ball; a basketball and then a tennis ball; or a ping-pong ball, a bowling ball, and then a basketball.

23. Given that you can only move one unit up or one unit to the right, how many ways are there to get to $(3, 3)$ from the origin without passing $(1, 1)$?

24. There exist four numbers such that taking the sums of the numbers in triplets gives the sums $3, 4, 5, 6$. Compute the sum of the squares of these four numbers.

25. A lattice point is a point whose coordinates are both integers. How many lattice points are on the boundary or inside the region bounded by $y = |x|$ and $y = -x^2 + 6$?

26. What is the sum $101 \cdot 100 - 100 \cdot 99 + 99 \cdot 98 - 98 \cdot 97 + \cdots + 3 \cdot 2 - 2 \cdot 1$?

27. Alicia is eating dried mangoes. The probability that the piece of mango is sweet is $\frac{2}{5}$ and the probability that it's sour is $\frac{3}{5}$. If the one she eats is sweet, she will eat another one. If the one she eats is sour, she will stop eating dried mangoes. If the expected number of dried mango pieces she will eat is in the form of $\frac{m}{n}$ and m, n are relatively prime, compute $m + n$.

28. Evaluate $\sqrt{12 - \sqrt{12 - \sqrt{12 - \sqrt{12 - \cdots}}}}$.

29. Let b, c be complex numbers, with a real, such that

$$a + b + c = ab + bc + ca = abc = 3.$$

and $a = m + \sqrt[k]{l}$, find $m + k + l$.

30. Find the sum of the roots of the equation $x^{45} + (1 - x)^{45} = 0$.

31. The math team captain wants to cut her 2-D circular watermelon to share with the 137 other people on the math team. If she can only make straight cuts that are chords of the circle, what is the smallest number of cuts she could make so that everyone on the math team gets a piece?

32. Ken has 101 bags of popcorn, which take $1, 2, \cdots 101$ minutes to make in a microwave. Luckily, Ken has two microwaves, meaning he can simultaneously have two bags of popcorn being made at the same time. Given that he can put in and take out popcorn from the microwaves at superhuman speed(ignore the time it takes to take and put in bags of popcorn), what's the minimum number of minutes it will take to make all the popcorn? (Once you put some bag of popcorn in a microwave, you can only take it out when it's done)

33. There are 100 empty coin pouches numbered 1 through 100 and 100 people numbered 1 through 100. Person n goes to every nth bag and drops n coins into it. For instance, the Person 1 goes to every pouch and drops 1 coin, Person 2 goes to every other pouch and drops 2 coins, and Person 3 goes to every third pouch and drops 3 coins, and so on. After all 100 people have gone, how many bags have an odd number of coins in them?

34. In trapezoid $ABCD$, $AB \parallel CD$ and $CD > AB$. Also, $AB = AC = AD = 5$ and $BC = 4$. If the length of BD is $a\sqrt{b}$, where b is square free, what is $a + b$?

35. Bobette only has red and blue socks and has 10 more red socks than blue socks. Not being able to see in her dark room, she randomly picks two socks. Oddly, she realizes that it is equally likely to get a pair of the same colored socks as different colored socks. How many blue socks does Bobette have?

36. Points E and F are randomly picked on the perimeter of rectangle $ABCD$, with $AB = CD = 20$ and $BC = DA = 37$. The probability that line segments AE and CF intersect can be written in the form $\frac{m}{n}$, for relatively prime positive integers m and n. Compute $m + n$.

37. Kelvin the Frog decides to play a game. He sets up a line of four lily pads in front of a dock. Every turn, he can jump to any of the lily pads in front of him, and he continues to jump until he hits the last lily pad. (He starts behind the first lilypad, so he may jump on to the first lilypad). For example, he might jump to the first lily pad, then the third, and then the fourth, totalling 3 turns. If the number of turns it will take him to reach the end, on average, is equal to $\frac{a}{b}$ for relatively prime integers a, b, what is $a + b$?

38. Justin is playing a game. There is a machine that has four identical balls, and will randomly select four squares on a six by six grid to put these balls on. Call an orientation "laughable" if no two balls lie in the same row or column, and no ball lies on the right of and above another ball. Justin wins the game if the orientation is "laughable." The probability that Justin wins can be expressed as $\frac{m}{n}$, where m and n are relatively prime. Find $m + \frac{n}{11}$.

39. Let circle ω_A be internally tangent to circle ω_B at a point C(internally tangent means that ω_A meet ω_B at one point, with the intersection of the interiors of the two circles being non-empty). Suppose that ω_B has area 25π and ω_A has area 9π. Let D be an arbitrary point on ω_A and E be an arbitrary point on ω_B. The maximum possible value of the area of $\triangle CDE$ can be written as $\frac{m\sqrt{n}}{p}$, where m and p are relatively prime positive integers, and n is a positive integer not divisible by the square of any prime. Find $m + n + p$.

40. A certain nation that is a collection of islands can be modeled on a coordinate plane: at each point $(2^m, 2^n)$ for non-negative integers m, n, there is a circular island of radius $\frac{1}{2^{m+n+1}}$ there. What is the average amount of coastline per unit area of this nation?