Joe Holbrook Memorial Invitational Competition (JHMIC)

4th Grade

March 28, 2021

General Rules

- You will have **90 minutes** to solve **16 questions**. Your score is the **sum** of the **point values** of the questions for which you got a correct answer. There are a total of 100 points.

  - Questions **1-5** are each worth **5 points**
  - Questions **6-10** are each worth **6 points**
  - Questions **11-13** are each worth **7 points**
  - Questions **14-16** are each worth **8 points**

- Only answers recorded on the appropriate Google Form will be graded.

- You are to remain visible to your proctor at **all times**. Please have your video camera on during the exam.

- This is an individual test. Anyone caught communicating with another student or using technology in an inappropriate way will be removed from the exam.

- You may not use the following aids:
  - Calculator or other computing device
 - Compass
  - Protractor
  - Ruler or straightedge

Other Notes

- All answers are positive integers. Please enter them with no spaces in between into the Google Form.

- Do not include commas in your answers. For example, the number one thousand is to be entered 1000 not $1,000$.

- You must not write units in your answers.

- Ties will be broken by the number of correct responses to questions 9 through 16. Further ties will be broken by the number of correct responses in the last four questions.

1. *[5 pts]* Compute $3(14 + 12) + 7(15 + 11)$.

2. *[5 pts]* When choosing his outfit, Nikhil has three choices for a shirt, four choices for pants, and five choices for his hat. However, he doesn't like 10 percent of the possible outfits he can create. Of the outfits he can create, how many does Nikhil like?

3. *[5 pts]* Find the smallest prime such that 2 plus that prime is not a prime or a square.

4. *[5 pts]* A polygon is formed by taking square $ABCD$ and gluing on an equilateral triangle that shares side AB with the square, then gluing on a square that shares side BC with the square, then gluing on a regular pentagon that shares side CD with the square, and finally a regular hexagon is glued on sharing side DA with the square. None of the shapes overlap with each other and are all on the same plane. How many sides does the polygon formed have?

5. *[5 pts]* A fair six sided die with the numbers 1 through 6 on its sides is rolled twice. The probability that the sum of the numbers rolled is prime is $\frac{a}{b}$ in simplest terms. What is $a + b$?

6. *[6 pts]* Fill in the other seven numbers in the following 3 by 3 grid with the numbers from 1-9 excluding 3 and 9 such that the sum of the numbers in each row is equal and the sum of the numbers in each column is equal and each number is used exactly once. Let a be the product of the four numbers in the corners and b be the sum of the four numbers in the corners. What is $a + b$?
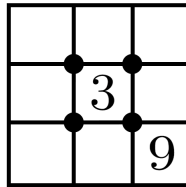
7. *[6 pts]* Two sides of a right triangle are 5 and 2. What is the minimum posssible value of the square of the area?

8. *[6 pts]* Three cows and two sheep can paint a barn in the same amount of time as a cow and seven sheep. Given that a cow can paint the barn in 5 hours, how many minutes will it take two cows and five sheep to paint the barn?

9. *[6 pts]* Let $S(k)$ be the number of ways to write k as the sum of two primes (the order of the primes do not matter). Compute the value of:

$$S(1) + S(3) + S(5) + \cdots + S(17) + S(19)$$

10. *[6 pts]* Nikhil and Jaiden are in a knockout style tournament with two other people. Initially, people are paired up against a random opponent and each person has a $\frac{1}{2}$ chance of winning and moving on the next round. The probability that Nikhil and Jaiden meet in the final and Nikhil wins can be written as $\frac{a}{b}$ in simplest form. What is $a + b$? The diagram below illustrates what a knockout style tournament may look like.
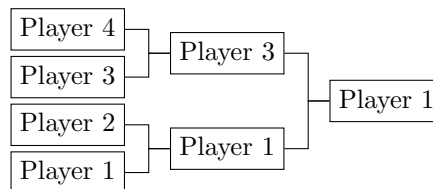
11. *[7 pts]* A set of distinct positive integers has the property that exactly half aren't divisible by 6, exactly a third aren't divisible by 2, and exactly a sixth aren't divisible by 3. What is the smallest possible sum of the elements of this set?

12. *[7 pts]* The set of five distinct numbers $\{a, b, c, d, e\}$ satisfy the property that the mean of the medians of the five sets $\{a, b, c, d\}, \{a, b, c, e\}, \{a, b, d, e\}, \{a, c, d, e\}$, and $\{b, c, d, e\}$ is equal to 1.2 times the median of the five means of these same sets. Given that the median of the original set is 51, what is the sum of its maximum and minimum element?

13. *[7 pts]* Let $ABCD$ be a square of side length 70 and point P be inside the square such that the ratio of the area of $\triangle APB$ to $\triangle CPD$ is 3:11 and the ratio of $\triangle BPC$ to $\triangle DPA$ is 5:2. What is AP?

14. *[8 pts]* How many ways are there to fill a 3 by 3 square with four 2 by 1 segments and a 1 by 1 square so that the grid is completely filled with no overlaps? The four 2 by 1 segments are indistinguishable.

15. *[8 pts]* If a has 8 positive divisors and b has 12 positive divisors, what is the smallest possible number of divisors that ab can have?

16. *[8 pts]* Jaiden swims with speed r. When swimming in the Lazy River that has no current, Jaiden swims 1 mile downstream and then 1 mile upstream in time $t_{\text{no current}}$. He then swims 1 mile downstream and then 1 mile back upstream in the Boppity river flowing at rate s downstream and that takes him time t_{current}. Given that the ratio between $t_{\text{no current}}$ and t_{current} is $120 : 121$, how many times faster is Jaiden swimming speed r than the current of Boppity river?