Joe Holbrook Memorial Invitational Competition (JHMIC)

## 5th Grade

March 28, 2021

General Rules

- You will have **90 minutes** to solve **16 questions**. Your score is the **sum** of the **point values** of the questions for which you got a correct answer. There are a total of 100 points.

    – Questions **1-5** are each worth **5 points**
    – Questions **6-10** are each worth **6 points**
    – Questions **11-13** are each worth **7 points**
    – Questions **14-16** are each worth **8 points**

- Only answers recorded on the appropriate Google Form will be graded.

- You are to remain visible to your proctor at **all times**. Please have your video camera on during the exam.

- This is an individual test. Anyone caught communicating with another student or using technology in an inappropriate way will be removed from the exam.

- You may not use the following aids:
    – Calculator or other computing device
    – Compass
   – Protractor
    – Ruler or straightedge

Other Notes

- All answers are positive integers. Please enter them with no spaces in between into the Google Form.

- Do not include commas in your answers. For example, the number one thousand is to be entered 1000 not $1,000$.

- You must not write units in your answers.

- Ties will be broken by the number of correct responses to questions 9 through 16. Further ties will be broken by the number of correct responses in the last four questions.

1. *[5 pts]* Compute $(2021)(2 + 0 + 2 + 1) + (2^2 + 2^0 + 2^2 + 2^1) \cdot (2^{2+0+2+1})$.

2. *[5 pts]* Find the smallest prime such that 2 plus that prime is not a prime or a square.

3. *[5 pts]* Fill in the other seven numbers in the following 3 by 3 grid with the numbers from 1-9 excluding 3 and 9 such that the sum of the numbers in each row is equal and the sum of the numbers in each column is equal and each number is used exactly once. Let a be the product of the four numbers in the corners and b be the sum of the four numbers in the corners. What is $a + b$?
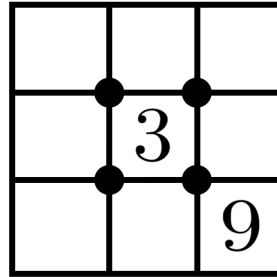

4. *[5 pts]* Two sides of a right triangle are 5 and 2. What is the minimum possible value of the square of the area?

5. *[5 pts]* A teacher with $n \geq 20$ students notices that for every prime p between 10 and 20, if she splits her students into groups of p until no more groups can be formed, a prime number of students will be left over. What's the smallest possible value of n?

6. *[6 pts]* A random 6 letter word composed of 2 A's, 2 B's and 2 C's is written. The probability that exactly 1 pair of identical letters are adjacent to each other is $\dfrac{p}{q}$ in simplest terms. What is $p + q$?

7. *[6 pts]* Three cows and two sheep can paint a barn in the same amount of time as a cow and seven sheep. Given that a cow can paint the barn in 5 hours, how many minutes will it take two cows and five sheep to paint the barn?

8. *[6 pts]* For a point P strictly inside the equilateral triangle $\triangle ABC$, segments PA, PB, and PC are drawn, determining angles $\angle APB, \angle BPC$, and $\angle CPA$ What is the closest integer to the greatest possible value for the positive difference between the mean and the median of these three angles?

9. *[6 pts]* Let a, b, and c be distinct integers such that $1 \leq a, b, c \leq 9$. How many triples (a, b, c) satisfy the property that the units digits of $a + b + c$ matches the units digit of at least one of $a, b,$ or c?

10. *[6 pts]* Nikhil and Jaiden are in a knockout style tournament with two other people. Initially, people are paired up against a random opponent and each person has a $\dfrac{1}{2}$ chance of winning and moving on the next round. The probability Nikhil and Jaiden meet in the final and Nikhil wins can be written as $\dfrac{a}{b}$ in simplest form. What is $a + b$? The diagram below illustrates what a knockout style tournament may look like.
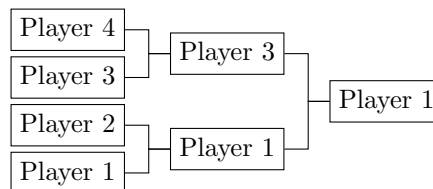
11. *[7 pts]* Jaiden swims with speed r. When swimming in the Lazy River that has no current, Jaiden swims 1 mile downstream and then 1 mile upstream in time $t_{\text{no current}}$. He then swims 1 mile downstream and then 1 mile back upstream in the Boppity river flowing at rate s downstream and that takes him time t_{current}. Given that the ratio between $t_{\text{no current}}$ and t_{current} is $120 : 121$, how many times faster is Jaiden swimming speed r than the current of Boppity river?

12. *[7 pts]* For real numbers a less than r, both with absolute value less than 1, the infinite geometric series with first term a and common ratio r has the same sum as the one with first term r and common ratio a. What is $2020(r + a)$?

13. *[7 pts]* How many triples of positive integers (x, y, z) satisfy $xyz + yz + z = 36$?

14. *[8 pts]* Greg writes the numbers 1, 2, 3, 4, 5 onto a blackboard. He writes the expression $(ab + c)d + e$ onto the board. He tells Nikhil to randomly substitute in the numbers into the five variables such that every number is used. The probability Nikhil's number is even can be written in the form $\dfrac{x}{y}$ in simplest terms. What is $x + y$?

15. *[8 pts]* A circle of radius 50 has two smaller circles of radius 24 that are internally tangent to it and externally tangent to each other. The bigger circle has a chord that is externally tangent to both smaller circles. What is the length of this chord?
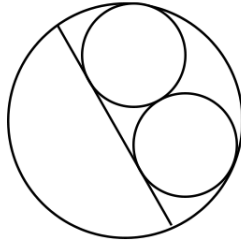
16. *[8 pts]* Let a, b, and c be the numbers $360 = 2^3 \cdot 3^2 \cdot 5^1, 1350 = 2^1 \cdot 3^3 \cdot 5^2$, and $1500 = 2^2 \cdot 3^1 \cdot 5^3$ in some order. What is the biggest possible value the following expression can take?

$$\gcd(\operatorname{lcm}(a, b), \operatorname{lcm}(b, c)) + \operatorname{lcm}(\gcd(a, b), \gcd(b, c))$$