

# Joe Holbrook Memorial Invitational Competition (JHMIC)

6th Grade

March 28, 2021

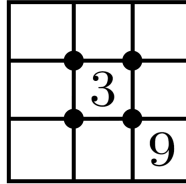
## General Rules

- You will have **90 minutes** to solve **16 questions**. Your score is the **sum** of the **point values** of the questions for which you got a correct answer. There are a total of 100 points.
  - Questions **1-5** are each worth **5 points**
  - Questions **6-10** are each worth **6 points**
  - Questions **11-13** are each worth **7 points**
  - Questions **14-16** are each worth **8 points**
- Only answers recorded on the appropriate Google Form will be graded.
- You are to remain visible to your proctor at **all times**. Please have your video camera on during the exam.
- This is an individual test. Anyone caught communicating with another student or using technology in an inappropriate way will be removed from the exam.
- You may not use the following aids:
  - Calculator or other computing device
  - Compass
  - Protractor
  - Ruler or straightedge

## Other Notes

- All answers are positive integers. Please enter them with no spaces in between into the Google Form.
- Do not include commas in your answers. For example, the number one thousand is to be entered 1000 not 1,000.
- You must not write units in your answers.
- Ties will be broken by the number of correct responses to questions 9 through 16. Further ties will be broken by the number of correct responses in the last four questions.

- [5 pts] What is  $4 + 41 \cdot 45 + 2 \cdot 41 + 2 \cdot 45$ ?
- [5 pts] Help Nikhil fill in the other seven numbers in the following 3 by 3 grid with the numbers from 1-9 such that the sum of the numbers in each row is equal and the sum of the numbers in each column is equal. (Note that 3 and 9 are already filled in and each number should be used exactly once). Let  $a$  be the product of the four numbers in the corners and  $b$  be the product of the four numbers on the sides. What is  $a + b$ ?

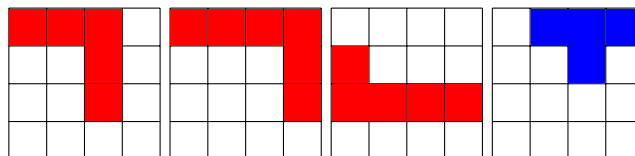


- [5 pts] Let  $S(k)$  be the number of ways to write  $k$  as the sum of two primes (the order of the primes do not matter). Compute the value of:

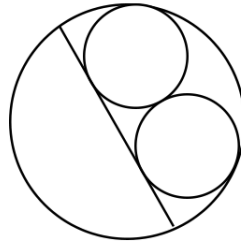
$$S(1) + S(3) + S(5) + \cdots + S(17) + S(19)$$

- [5 pts] A random 6 letter word composed of 2 A's, 2 B's and 2 C's is written. The probability that exactly 1 pair of identical letters are adjacent to each other is  $\frac{p}{q}$  in simplest terms. What is  $p + q$ ?
- [5 pts] A set of distinct positive integers has the property that exactly half aren't divisible by 6, exactly a third aren't divisible by 2, and exactly a sixth aren't divisible by 3. What is the smallest possible sum of the elements of this set?
- [6 pts] Rectangle  $ABCD$  with  $AB = 4$  and  $BC = 8$  is reflected over  $BD$ . What is the area of the union of  $ABCD$  and its reflection?
- [6 pts] The set of five distinct numbers  $\{a, b, c, d, e\}$  satisfy the property that the mean of the medians of the five sets  $\{a, b, c, d\}$ ,  $\{a, b, c, e\}$ ,  $\{a, b, d, e\}$ ,  $\{a, c, d, e\}$ , and  $\{b, c, d, e\}$  is equal to 1.2 times the median of the five means of these same sets. Given that the median of the original set is 51, what is the sum of its maximum and minimum element?
- [6 pts] Let  $ABCD$  be a square of side length 70 and point  $P$  be inside so that the ratio of the area of  $APB$  to  $CPD$  is 3:11 and the ratio of  $BPC$  to  $DPA$  is 5:2. What is  $AP$ ?
- [6 pts] Let there be a 3 by 4 grid. Define a  $L$ -shape as the union of a 1 by  $m$  strip and a  $n$  by 1 strip such that
  - the two strips are perpendicular
  - the two strips share one cell that is an endpoint of both strips
  - both strips have a length of at least 2

How many  $L$ -shapes are there in the grid? The red shapes below illustrate some valid  $L$ -shapes, while the blue shape is not a valid  $L$ -shape.



- [6 pts] In how many ways can Jim distribute 180 indistinguishable apples amongst his three friends, Al, Bob, and Chrysanthemum, such that the number of apples that each person receives is a factor of the number of apples that someone else receives? For example, a configuration that works is Al receiving 4 apples, with Bob and Chrysanthemum receiving 88 apples each.
- [7 pts] Greg writes the numbers 1, 2, 3, 4, 5 onto a blackboard. He writes the expression  $(ab + c)d + e$  onto the board. He tells Nikhil to randomly substitute in the numbers into the five variables such that every number is used. The probability Nikhil's number is even can be written in the form  $\frac{x}{y}$  in simplest terms. What is  $x + y$ ?



12. [7 pts] For real numbers  $a$  less than  $r$ , both with absolute value less than 1, the infinite geometric series with first term  $a$  and common ratio  $r$  has the same sum as the one with first term  $r$  and common ratio  $a$ . What is  $2020(r + a)$ ?
13. [7 pts] A circle of radius 50 has two smaller circles of radius 24 that are internally tangent to it and externally tangent to each other. The bigger circle has a chord that is externally tangent to both smaller circles. What is the length of this chord?
14. [8 pts] If  $a$  has 8 positive divisors and  $b$  has 12 positive divisors, what is the sum of the smallest and largest possible number of divisors that  $ab$  can have?
15. [8 pts] Compute the sum of the expression  $\frac{(5^k + 4^k)(5^k - 4^k)}{400^k}$  over all positive integers  $k$ .
16. [8 pts] For positive integers  $a$  and  $b$ , Greg chooses a random integer between 1 and  $a$  inclusive while Nikhil chooses a random integer between 1 and  $b$  inclusive and they find that the expected product of their numbers is equal to three times the expected sum. What is the sum of all possible values of  $a$ ?