Joe Holbrook Memorial Invitational Competition (JHMIC)

7th Grade

March 28, 2021

General Rules

- You will have **90 minutes** to solve **16 questions**. Your score is the **sum** of the **point values** of the questions for which you got a correct answer. There are a total of 100 points.
  - Questions **1-5** are each worth **5 points**
  - Questions **6-10** are each worth **6 points**
  - Questions **11-13** are each worth **7 points**
  - Questions **14-16** are each worth **8 points**
- Only answers recorded on the appropriate Google Form will be graded.
- You are to remain visible to your proctor at **all times**. Please have your video camera on during the exam.
- This is an individual test. Anyone caught communicating with another student or using technology in an inappropriate way will be removed from the exam.
- You may not use the following aids:
  - Calculator or other computing device
  - Compass
  - Protractor
  - Ruler or straightedge

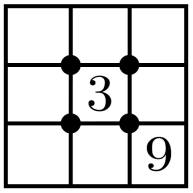
Other Notes

- All answers are positive integers. Please enter them with no spaces in between into the Google Form.
- Do not include commas in your answers. For example, the number one thousand is to be entered 1000 not $1,000$.
- You must not write units in your answers.
- Ties will be broken by the number of correct responses to questions 9 through 16. Further ties will be broken by the number of correct responses in the last four questions.

1. *[5 pts]*A fair six sided die with the numbers 1 through 6 on its sides is rolled twice. The probability that the sum of the numbers rolled is prime is $\frac{a}{b}$ in simplest terms. What is $a+b$?

2. *[5 pts]* For how many integers b is 114_b a base b number that is less than 1000?

3. *[5 pts]* Help Nikhil fill in the other seven numbers in the following 3 by 3 grid with the numbers from 1-9 such that the sum of the numbers in each row is equal and the sum of the numbers in each column is equal. (Note that 3 and 9 are already filled in and each number should be used exactly once). Let a be the product of the four numbers in the corners and b be the product of the four numbers on the sides. What is $a+b$?



4. *[5 pts]* Three cows and two sheep can paint a barn in the same amount of time as a cow and seven sheep. Given that a cow can paint the barn in 5 hours, how many minutes will it take two cows and five sheep to paint the barn?

5. *[5 pts]* Let $ABCD$ be a square of side length 70 and point P be inside the square such that the ratio of the area of $\triangle APB$ to $\triangle CPD$ is 3:11 and the ratio of $\triangle BPC$ to $\triangle DPA$ is 5:2. What is AP?

6. *[6 pts]* The five distinct numbers a, b, c, d, e satisfy the property that the mean of the medians of the five sets $\{a, b, c, d\}, \{a, b, c, e\}, \{a, b, d, e\}, \{a, c, d, e\}$, and $\{b, c, d, e\}$ is equal to 1.2 times the median of the five means of these same sets. Given that the median of the original set is 51, what is the sum of its maximum and minimum element?

7. *[6 pts]* In how many ways can Jim distribute 180 indistinguishable apples amongst his three friends, Al, Bob, and Chrysanthemum, such that the number of apples that each person receives is a factor of the number of apples that someone else receives? For example, a configuration that works is Al receiving 4 apples, with Bob and Chrysanthemum receiving 88 apples each.

8. *[6 pts]* Let there be a clock with two hands, one with length 2 and one with length 1. Alice picks a random time to look at the clock. The probability the area of the triangle formed by the two clock hands at that time is at least $\frac{1}{2}$ can be written as $\frac{a}{b}$, where a, b have a greatest common divisor of 1. What is $a+b$?

9. *[6 pts]* Greg writes the numbers 1, 2, 3, 4, 5 onto a blackboard. He writes the expression $(ab+c)d+e$ onto the board. He tells Nikhil to randomly substitute in the numbers into the five variables such that every number is used. The probability Nikhil's number is even can be written in the form $\frac{x}{y}$ in simplest terms. What is $x+y$?

10. *[6 pts]* For real numbers a less than r, both with absolute value less than 1, the infinite geometric series with first term a and common ratio r has the same sum as the one with first term r and common ratio a. If $r \cdot a = \frac{10}{49}$, then $2020(r-a)$ is in simplest terms $\frac{p}{q}$. What is p+q?

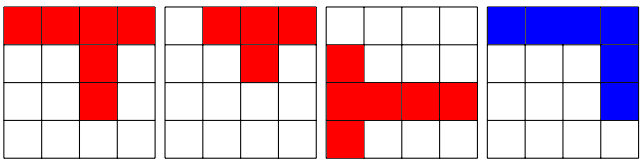
11. *[7 pts]* Let there be a 4 by 4 grid. Define a T-shape as the union of a 1 by m strip and a n by 1 strip such that

    (a) the two strips are perpendicular

    (b) the two strips share one cell that is the endpoint of one of the strips but not the other

    (c) both strips have a length of at least 2

    How many T-shapes are there in the grid? The red shapes below illustrate some valid T-shapes, while the blue shape is not a valid T-shape.



12. *[7 pts]* What is the smallest positive palindrome that is divisible by 7, 11, and 19?

13. *[7 pts]* For positive integers a and b, Greg chooses a random integer between 1 and a inclusive while Nikhil chooses a random integer between 1 and b inclusive and they find that the expected product of their numbers is equal to three times the expected sum. What is the sum of all possible values of a?

14. *[8 pts]* If Jaiden starts writing his essay at 10:30 he has a 100 percent chance of getting in to PROSS. Let p_i represent the probability he gets in after procrastinating i minutes. If

$$p_i + \frac{3}{p_i} = p_{i+1} + \frac{3}{p_{i+1}} - \frac{1}{20}$$

for $i \geq 0$, what is the maximum amount of integer minutes he can procrastinate and have at least a 60% of getting in?

15. *[8 pts]* Let N denote the sum of the divisors of 700000 that have a remainder of 1 when divided by 3. Compute the sum of the distinct prime factors of N.

16. *[8 pts]* For non-degenerate $\triangle ABC$, let $f(\triangle ABC)$ be the number of intersection points in the diagram formed by drawing $\triangle ABC$, the circle with diameter AB, the circle with diameter AC, and the circle with diameter BC. The average value of $f(\triangle ABC)$ over all non-congruent triangles ABC with distinct integer sides between 20 and 30 inclusive can be expressed in simplest terms as $\frac{x}{y}$. What is $x + y$?