

Joe Holbrook Memorial Math Competition

4th Grade

October 17, 2021

General Rules

- You will have **75 minutes** to solve **40 questions**. Your score is the number of correct answers.
- Only answers recorded on the answer sheet will be graded.
- This is an individual test. Anyone caught communicating with another student will be removed from the exam and their score will be disqualified.
- Scores will be posted on the website. Please do not forget your ID number, as that will be the sole means of identification for the scores.
- You may use the following aids:
 - Pencil or other writing utensil
 - Eraser
 - Blank scrap paper
- You may not use the following aids:
 - The Internet
 - Books or other written sources
 - Other people
 - Calculator or other computing device
 - Compass
 - Protractor
 - Ruler or straightedge

Other Notes

- Please input your answers into the Google form provided by your proctor.
- All answers are integers. Make sure you do not make any typing mistakes, as you will not be given credit if you do so.
- You do not need to write units in your answers.
- Ties will be broken by the number of correct responses to questions 31 through 40. Further ties will be broken by the number of correct responses in the last five questions.

1. What is $2021 - 2120 + 1202$?
2. What is $(7 + 7) \times ((7 \div 7) \div 7) + (7 - 7)^7$?
3. A square has a perimeter of 40. What is the area of the square?
4. Bianca is counting the number of windows she has. There are 3 times as many windows on her family PC as there are on her Mac. If she has 20 windows in total across her two computers, how many windows does she have on her PC?
5. A bag has 90 beads. 45 are red, 30 are green, and the rest are blue. If the ratio of blue beads to red beads is $\frac{a}{b}$ in simplest form, what is $a + b$?
6. There were 101 dalmatians but 34 go missing. Some of the left over dalmatians are split into 5 equal groups so that the groups are as big as possible. How many dalmatians are not missing and are not in a group?
7. Each day, Catherine drives four miles more than she drove the previous day. On the first day, Catherine drove twelve miles. How many miles will Catherine drive on the fifth day?
8. If one dollar is 100 cents, then how many cents is one half of two thirds of three quarters of four fifths of a dollar?
9. Jennifer the rat is trying to run away from Ashley the cat who is chasing her! They are currently 50 meters apart. If Jennifer runs at 3 meters per second and Ashley runs at 13 meters per second, how many seconds will it take for Jennifer to be caught?
10. Erez has a water bottle filled with 32 ounces of water. He drinks x ounces of water from the water bottle. Then he drinks $\frac{1}{3}$ of the remaining water in the water bottle. He now has 16 ounces of water left. What is x ?
11. Square $ABCD$ has area 16. If E lies outside the square such that ADE is an equilateral triangle, find the perimeter of pentagon $ABCDE$.
12. Twenty kids in a classroom take turns using a pencil sharpener. If each kid takes 5 seconds to sharpen his or her pencil and 2 seconds to switch turns with the person behind them, how long will it take for the entire class to sharpen their pencils? (Each kid has exactly one pencil.)
13. Both Pineapple and Jamsung are advertising their new charging rates for their phones! Pineapple's iPine charges 15% per hour, while Jamsung's Jalaxy charges at 1% every 6 minutes. Find the difference between the amount of time in minutes it takes for an iPine and a Jalaxy device to charge to 100%.
14. Yul is often late when meeting up with Alicia. They agree to meet at 5pm, but there is only a 20% probability that Yul arrives by 5pm. There is a 20% probability that she arrives after 5pm but before 5:10pm, a 30% probability that she arrives after 5:10pm but before 5:20pm, and a 30% probability that she arrives after 5:20pm but before 5:30pm. How many minutes late should Alicia arrive so that there is a 70% chance Yul will arrive by that time?
15. Two friends, Mikey and Rohit, arrive at a circular park. The two begin to walk along the perimeter of the park in opposite directions of each other at the exact same speed, meeting at the opposite end of the circle at the same time after each walking 18π miles. If they then decide to walk together in a straight pathway back to their point of arrival, how many more miles would each person have to walk?
16. Tommy is playing basketball with his friend Toby, but he is very bad at it! Out of the 12 shots thrown, only 3 make it in. However, Toby is much better, with a success percentage triple of Tommy's. If Toby makes 16 attempts, how many baskets does he successfully make?
17. In a comedy talent show, a panel of three judges categorize a performance as "hilarious" if they all rate a performance between a 7 and 10, inclusive. Additionally, they categorize a performance as "embarrassing" if they all rate a performance between a 1 and 3, inclusive. Assuming each judge gives an integer score from 1 to 10, inclusive, how many possible total sums of scores are there for performances that are either hilarious or embarrassing?
18. The answer to this question is eleven more than the number of digits of the answer to this question. What is the answer to this question?

19. Satwika is eating a circular cookie of radius 10 cm. One fourth of the total area of the cookie is covered by non-overlapping circular chocolate chips of radius 1 cm. How many chocolate chips are on Satwika's cookie?
20. Lili has a list of all the positive multiples of 6 less than or equal to 100. Zoey has a list of all the positive integers less than or equal to 100 that are 1 greater than a multiple of 4. If Lili's list has L numbers and Zoey's list has Z numbers, what is $L - Z$?
21. Rectangle $ABCD$ has side lengths 3 and 5. Point E is drawn on AB and lines CE and DE are also drawn. The area of $\triangle DEC$ can be written as $\frac{a}{b}$. Compute $a + b$.
22. 4 distinct integers $a, b, c,$ and d have a product of 25. What is $a^2 + b^2 + c^2 + d^2$?
23. What is the smallest prime number such that $p - 1$ and $p + 1$ have the same number of divisors?
24. Kelvin the Frog is tiling his home, a rectangle with sides of length 6 yards by 8 yards, with 108 perfect circular lilypads of radius 1 foot (there are 3 feet in a yard). If none of the lilypads are overlapping or cut off, and the area of the pond that is uncovered in square feet can be expressed as $n - m\pi$, what is $n + m$?
25. Rosie tells Jennie that her favorite number is a positive integer less than 100 which has an odd number of positive divisors. Jennie then asks what the units digit of Rosie's favorite number is. After Rosie answers, Jennie immediately knows Rosie's favorite number. What is Rosie's favorite number?
26. Nikhil has a potato farm that needs to be farmed. He can hire x clones of Jaiden and y clones of Lance. If a clone of Jaiden can farm $\frac{1}{1700}$ of the farm per day, and a clone of Lance can farm $\frac{1}{3400}$ of the farm per day, and Nikhil wishes to farm all the potatoes in exactly one day, how many ways can he hire x clones of Jaiden and y clones of Lance? (Clones are indistinguishable from each other).
27. Cookie Monster wants to buy a cookie for 67 cents. If he has 5 of each of the pennies, nickels, dimes and quarters, then find the number of ways he can pay using at least 3 dimes.
28. Harry has a box which contains 12 blue socks, 13 red socks, and 15 green socks. How many socks must Harry pick out of the box to guarantee that he has picked out at least one pair of red socks?
29. Jaiden has a circular corn farm with radius 20 miles. For some rather peculiar reason, Jaiden puts an enormous rectangle on his farm, with one side length equal to 32 miles, and where each corner of the rectangle is on the circumference of the farm. In the rectangle, he builds a humongous swimming pool! Jaiden now has $n\pi - m$ square miles of land to use to grow corn for positive integers n and m . Jaiden cannot grow corn in the swimming pool. What is $n + m$?
30. Maui is between a cave and the shore. He wants to get to the shore and could do so by running directly to it. Alternatively, he could run to the cave, instantly grab his hook and transform into a bird, then fly to shore. He flies 4 times faster than he walks, and both choices require the same amount of time. If the ratio of Maui's distance to the shore to his distance to the cave is $a : b$ in lowest terms, find $a + b$.
31. What is the smallest 5-digit number \underline{abcde} such that $a < b, b > c, c < d,$ and $d > e$? a, b, c, d, e are distinct digits and $a \neq 0$.
32. The side lengths of a triangle are 2cm, 4cm, and 3cm. One of the side lengths of a similar triangle is 12cm. What is the sum of the possible perimeters of this other triangle, in cm?
33. How many positive perfect cubes divide $3! \cdot 5! \cdot 8!$?
34. There are 4 indistinguishable red books, 3 indistinguishable blue books, and 2 indistinguishable green books on a shelf. In how many ways can they be ordered?
35. An octahedron is an 8-sided 3D shape created by attaching two square pyramids together by their square bottoms. What is the minimum number of paint colors needed to paint an octahedron so that no two adjacent faces are the same color?
36. Kelvin the Frog wants to color each of the first 21 positive integers either red or blue. He wants to do this in such a way that an integer and twice that integer are not the same color. How many different colorings are possible?

37. Square $ABCD$ has side length 1. Point E is chosen on the outside of the square so that ABE is an isosceles right triangle with hypotenuse AB . If DE can be written in the form $\sqrt{\frac{a}{b}}$, where $\gcd(a, b) = 1$, find $a + b$.
38. At his national karate competition, Nikhil must win against 3 opponents in a row in order to be *satisfied*. Every match, Nikhil has a $\frac{1}{3}$ chance of winning. The probability that Nikhil becomes satisfied if he is scheduled for 5 matches is $\frac{m}{n}$ for relatively prime integers m and n . What is $m + n$?
39. A parabola $y = 3x^2 + 2x - 1$ goes through a translation where every point (x, y) on the parabola is moved to the point $(x + a, y + b)$. The new equation for the parabola is $y = 3x^2 - 10x + 12$. What is $a + b$?
40. Jasmine and Aladdin decide to meet up at night but don't want to risk getting caught. They decide that each of them should wait at a meeting spot for half an hour any time between midnight and 3:00 AM. If the probability they will meet each other is $\frac{a}{b}$ where a and b are relatively prime integers, what is $b - a$?