

Joe Holbrook Memorial Math Competition

5th Grade

October 17, 2021

General Rules

- You will have **75 minutes** to solve **40 questions**. Your score is the number of correct answers.
- Only answers recorded on the answer sheet will be graded.
- This is an individual test. Anyone caught communicating with another student will be removed from the exam and their score will be disqualified.
- Scores will be posted on the website. Please do not forget your ID number, as that will be the sole means of identification for the scores.
- You may use the following aids:
 - Pencil or other writing utensil
 - Eraser
 - Blank scrap paper
- You may not use the following aids:
 - The Internet
 - Books or other written sources
 - Other people
 - Calculator or other computing device
 - Compass
 - Protractor
 - Ruler or straightedge

Other Notes

- Please input your answers into the Google form provided by your proctor.
- All answers are integers. Make sure you do not make any typing mistakes, as you will not be given credit if you do so.
- You do not need to write units in your answers.
- Ties will be broken by the number of correct responses to questions 31 through 40. Further ties will be broken by the number of correct responses in the last five questions.

1. What is $(7 + 7) \times ((7 \div 7) \div 7) + (7 - 7)^7$?
2. Bianca is counting the number of windows she has. There are 3 times as many windows on her family PC than there are on her Mac. If she has 20 windows in total across the two computers, how many windows does she have on her family PC?
3. Compute $(2 + 0 + 2 + 1)^2 + 2^{(2+0+2+1)}$.
4. Each day, Catherine drives four miles more than the previous day. On the first day, Catherine drove twelve miles. How many miles will Catherine drive on the fifth day?
5. There were 101 dalmatians but 34 go missing. Some of the left over dalmatians are split into 5 equal groups so that the groups are as big as possible. How many dalmatians are not missing and are not in a group?
6. If one dollar is 100 cents, then how many cents is one half of two thirds of three quarters of four fifths of a dollar?
7. What is the smallest prime number such that $p - 1$ and $p + 1$ have the same number of divisors?
8. Twenty kids in a classroom take turns using a pencil sharpener. If each kid takes 5 seconds to sharpen their pencil and 2 seconds to switch turns to the person behind, how long will it take for the entire class to sharpen their pencils? (Each kid has exactly one pencil.)
9. Both Pineapple and Jamsung are advertising their new charging rates for their phones! Pineapple's iPine charges 15% per hour, while Jamsung's Jaxy charges at 1% every 6 minutes. Find the positive difference between the amount of time in minutes it takes for an iPine and a Jaxy device to charge to 100%.
10. Yul is often late when meeting up with Alicia. They agree to meet at 5pm, but there is only a 20% probability that Yul arrives by 5pm. There is a 20% probability that she arrives after 5pm but before 5:10pm, a 30% probability that she arrives after 5:10pm but before 5:20pm, and a 30% probability that she arrives after 5:20pm but before 5:30pm. How many minutes after 5pm should Alicia arrive so that there is a 70% chance Yul will arrive by that time?
11. Two friends, Mikey and Rohit, arrive at a circular park. The two begin to walk along the perimeter of the park in opposite directions at the exact same speed, meeting at the opposite end of the circle at the same time after each walking 18π miles. If they then decide to walk together in a straight pathway back to their point of arrival, how many more miles would each person have to walk?
12. Janice's age is currently half of John's age. If John is 24 years old now, how many years old will Janice be when John is 50 years old?
13. The answer to this question is eleven more than the number of digits of the answer to this question. What is the answer to this question?
14. Kelvin the Frog is tiling his home, a rectangle with sides of length 6 yards by 8 yards, with 108 perfect circular lilypads of radius 1 foot (there are three feet in a yard). If none of the lilypads are overlapping or cut off, the area of the pond that is uncovered in square feet can be expressed as $n - m\pi$, where n, m are positive integers. What is $n + m$?
15. Rosie tells Jennie that her favorite number is a positive integer less than 100 and has an odd number of positive divisors. Jennie then asks what the unit's digit of Rosie's favorite number is. After Rosie answers, Jennie immediately knows Rosie's favorite number. What is Rosie's favorite number?
16. In a comedy talent show, a panel of three judges categorize a performance as "hilarious" if they all rate a performance between a 7 and 10, inclusive. Additionally, they categorize a performance as "embarrassing" if they all rate a performance between a 1 and 3, inclusive. Assuming each judge gives an integer score from 1 to 10, inclusive, how many possible total sum of scores are there for performances that are either hilarious or embarrassing?
17. Nikhil has a potato farm that needs to be farmed. He can hire x clones of Jaiden and y clones of Lance. If a clone of Jaiden can farm $\frac{1}{1700}$ of the farm per day and Lance can farm $\frac{1}{3400}$ of the farm per day and Nikhil wishes to farm all the potatoes in exactly one day, how many ways can he hire x clones of Jaiden and y clones of Lance? (Clones are indistinguishable from each other).

18. Satwika is eating a circular cookie of radius 10 cm. One fourth of the total area of the cookie is covered by non-overlapping circular chocolate chips of radius 1 cm. How many chocolate chips are on Satwika's cookie?
19. Harry has a box which contains 12 blue socks, 13 red socks, and 15 green socks. How many socks must Harry pick out of the box to guarantee that he has picked out at least one pair of red socks?
20. Jaiden has a circular corn farm with radius 20 miles. For some peculiar reason, Jaiden puts an enormous rectangle on his farm, with one side length equal to 32 miles and where each corner of the rectangle is on the circumference of the farm. In the rectangle, he builds a humongous swimming pool! Jaiden now has $n\pi - m$ square miles of land to use to grow corn for positive integers n and m . Jaiden cannot grow corn in the swimming pool. What is $n + m$?
21. How many positive integers less than 80 are not divisible by 5 or 7?
22. Maui is between a cave and the shore. He wants to get to the shore and could do so by running directly to it. Alternatively, he could run to the cave, instantly grab his hook and transform into a bird, then fly to shore. He flies 4 times faster than he walks, and both choices require the same amount of time. If the ratio of Maui's distance to the shore to his distance to the cave is $a : b$ in lowest terms, find $a + b$.
23. In the strange world of Mumblejumbleria, each of the 10,000 aliens are named by a sequence of 3 letters. Find the smallest number of letters that can be used such that every alien has a different name.
24. What is the smallest 5-digit number \underline{abcde} such that $a < b, b > c, c < d$, and $d > e$? a, b, c, d, e are distinct digits and $a \neq 0$.
25. The side lengths of a triangle are 2cm, 4cm, and 3cm. One of the side lengths of a similar triangle is 12cm. What is the sum of the possible perimeters of this other triangle, in cm?
26. How many positive perfect cubes divide $3! \cdot 5! \cdot 8!$?
27. A broken calculator only has the functions of multiplying by 2 and adding 1. If Bob starts with the number 3, what is the least number of operations he must perform to reach 101?
28. A school of moonfish is swimming in a triangular formation. It so happens that the side lengths of the triangle are 29 cm, 29 cm, and 40 cm. Suddenly, a shark passes the school, so it breaks formation and rearrange itself. The new shape is a triangle with side lengths 29 cm, 29 cm, and x cm, where x is an integer not equal to 40. If the area of the triangle remains the same, what is the value of x ?
29. A palindrome is a number that reads the same forwards and backwards, such as 1551 or 38783. How many 5-digit palindromes are divisible by 4?
30. An ice cream truck offers 1 or 2 scoops of ice cream and toppings or no toppings. In a group of 20 kids that bought from the ice cream truck, 12 got 2 scoops and 4 got no toppings. If 2 kids got 2 scoops with no toppings, how many kids got one scoop with toppings?
31. Kelvin the Frog wants to color each of the first 21 positive integers either red or blue. He wants to do this in such a way that an integer and twice that integer are not the same color. How many different colorings are possible?
32. In $\triangle ABC$, \overline{AB} has side length 4. A point D is constructed on \overline{BC} , such that $CD = 6$. \overline{AC} is then extended until a point E, such that $\overline{AD} = \overline{DE}$, and $\angle DEC \cong \angle BAD$. What is the product of \overline{AC} and \overline{BD} ?
33. A parabola $y = 3x^2 + 2x - 1$ goes through a translation where every point (x, y) on the parabola is moved to the point $(x + a, y + b)$. The new equation for the parabola is $y = 3x^2 - 10x + 12$. What is $a + b$?
34. Jasmine and Aladdin decide to meet up at night but don't want to risk getting caught. They decide that each of them should wait at a meeting spot for half an hour any time between midnight and 3:00 AM. If the probability they will meet each other is $\frac{a}{b}$ where a and b are relatively prime integers, what is $b - a$?
35. Annie, Blake, Christina, Dong Joo, and Eric are waiting in line for jelly bean candies. Eric is standing somewhere ahead of Dong Joo, and Blake is not standing directly in front of or behind Annie. How many ways are there for them to be arranged in line?
36. Let $f(x) = x^3 + ax^2 + bx + c$ for integers a, b, c . If $f(1) = 1$ and $f(2) = 2$, but $f(x) \neq x$ for all other values of x , then what is the sum of the possible values of $f(0)$?

37. In equiangular hexagon $YULKIM$, let $YU = LK = IM = 4$ and $UL = KI = MY = 6$. The area of $YULKIM$ is denoted $[YULKIM]$. Find $[YULKIM]^2$.
38. Bobette only has red and blue socks and has 10 more red socks than blue socks. Not being able to see in her dark room, she randomly picks two socks. Oddly, she realizes that it is equally likely to get a pair of the same colored socks as different colored socks. How many blue socks does Bobette have?
39. Find the remainder when $3^{9^{27^{81}}}$ is divided by 7.
40. Nikhil loves numbers that consists of the digits 1, 3, and/or 9 so much that he makes his own sequence called the *Nikhili Numbers*. The i th Nikhili Number is the i th smallest positive integer consisting of 1s, 3s, or 9s **that does not start with a 1**. The i th Nikhili Number is denoted as n_i . For example, $n_1 = 3, n_2 = 9, n_3 = 31, n_4 = 33, n_5 = 39, n_6 = 91$. What is n_{2021} ?