Joe Holbrook Memorial Math Competition

6th Grade

October 17, 2021

General Rules

- You will have **75 minutes** to solve **40 questions**. Your score is the number of correct answers.
- Only answers recorded on the answer sheet will be graded.
- This is an individual test. Anyone caught communicating with another student will be removed from the exam and their score will be disqualified.
- Scores will be posted on the website. Please do not forget your ID number, as that will be the sole means of identification for the scores.
- You may use the following aids:
  - Pencil or other writing utensil
  - Eraser
  - Blank scrap paper
- You may not use the following aids:
  - The Internet
  - Books or other written sources
  - Other people
  - Calculator or other computing device
  - Compass
  - Protractor
  - Ruler or straightedge

Other Notes

- Please input your answers into the Google form provided by your proctor.
- All answers are integers. Make sure you do not make any typing mistakes, as you will not be given credit if you do so.
- You do not need to write units in your answers.
- Ties will be broken by the number of correct responses to questions 31 through 40. Further ties will be broken by the number of correct responses in the last five questions.

1. Bianca is counting the number of windows she has. There are 3 times as many windows on her family PC as there are on her Mac. If she has 20 windows in total across her two computers, how many windows does she have on her PC?

2. A bag has 90 beads. 45 are red, 30 are green, and the rest are blue. If the ratio of blue beads to red beads is $\frac{a}{b}$ in simplest form, what is $a+b$?

3. A rectangles has side lengths 4 and 5. Suppose that the side length of four is halved. What must the other side length be in order to maintain the area of the rectangle?

4. 5 years ago, Maryam was twice as old as Yuval. 11 years ago, Maryam was 3 times as old as Yuval. How old is Yuval now?

5. Two friends, Mikey and Rohit, arrive at a circular park. The two begin to walk along the perimeter of the park in opposite directions of each other at the exact same speed, meeting at the opposite end of the circle at the same time after each walking 18π miles. If they then decide to walk together in a straight pathway back to their point of arrival, how many more miles would each person have to walk?

6. Square $ABCD$ has area 16. If E lies outside the square such that ADE is an equilateral triangle, find the perimeter of pentagon $ABCDE$.

7. Cookie Monster wants to buy a cookie for 67 cents. If he has 5 of each of the pennies, nickels, dimes and quarters, then find the number of ways he can pay using at least 3 dimes.

8. Lili has a list of all the positive multiples of 6 less than or equal to 100. Zoey has a list of all the positive integers less than or equal to 100 that are 1 greater than a multiple of 4. If Lili's list has L numbers and Zoey's list has Z numbers, what is $L-Z$?

9. Alex asked 100 students if they own a red car, a blue car, or both. 15 students own both a red car and a blue car, and the number of people who own a red card is four times as great as the number of students owning a blue car. How many students own a blue car?

10. 4 distinct integers a, b, c, and d have a product of 25. What is $a^2+b^2+c^2+d^2$?

11. Christina has \$5.00 and goes to a store. If she spends all her money on soda, she can get at most 4 bottles, and she can similarly get at most 8 packs of gum if she spends all of her money on gum. Each bottle of soda costs the same, each pack of gum costs the same, and everything costs a whole number of cents. There is also no tax or any other extra cost. What is the highest price 3 bottles of soda and 6 packs of gum could be, in cents?

12. Rosie tells Jennie that her favorite number is a positive integer less than 100 which has an odd number of positive divisors. Jennie then asks what the units digit of Rosie's favorite number is. After Rosie answers, Jennie immediately knows Rosie's favorite number. What is Rosie's favorite number?

13. In a comedy talent show, a panel of three judges categorize a performance as "hilarious" if they all rate a performance between a 7 and 10, inclusive. Additionally, they categorize a performance as "embarrassing" if they all rate a performance between a 1 and 3, inclusive. Assuming each judge gives an integer score from 1 to 10, inclusive, how many possible total sums of scores are there for performances that are either hilarious or embarrassing?

14. Nikhil has a potato farm that needs to be farmed. He can hire x clones of Jaiden and y clones of Lance. If a clone of Jaiden can farm $\frac{1}{1700}$ of the farm per day, and a clone of Lance can farm $\frac{1}{3400}$ of the farm per day, and Nikhil wishes to farm all the potatoes in exactly one day, how many ways can he hire x clones of Jaiden and y of Lance? (Clones are indistinguishable from each other).

15. Satwika is eating a circular cookie of radius 10 cm. One fourth of the total area of the cookie is covered by non-overlapping circular chocolate chips of radius 1 cm. How many chocolate chips are on Satwika's cookie?

16. Jaiden has a circular corn farm with radius 20 miles. For some rather peculiar reason, Jaiden puts an enormous rectangle on his farm, with one side length equal to 32 miles, and where each corner of the rectangle is on the circumference of the farm. In the rectangle, he builds a humongous swimming pool! Jaiden now has $n\pi - m$ square miles of land to use to grow corn for positive integers n and m. Jaiden cannot grow corn in the swimming pool. What is $n+m$?

17. Maui is directly between a cave and his boat. He wants to get to his boat and could do so by running directly to it. Alternatively, he could run to the cave, grab his hook, transform into a bird, and fly to his boat. He flies 4 times faster than he walks, and both choices require the same amount of time. If the ratio of Maui's distance to the boat to his distance to the cave is $a : b$ in lowest terms, find $a + b$.

18. King Triton took a census of his whole kingdom, which are entirely merpeople, but forgot what the exact population was. The only thing he remembers is that the number was eight digits long and was formed by repeating a four-digit integer twice. For examples, some possible population numbers include $12,341,234$ and $81,478,147$. King Triton is holding a large festival and needs to know how to evenly split the population into an integer number of groups. As of now, what is the greatest number of merpeople he can put in each group?

19. Let $f(x) = x^2 + kx + 100$. For how many integer values of k in the interval $[-100, 100]$ will there be no real solutions for x?

20. The side lengths of a triangle are 2cm, 4cm, and 3cm. One of the side lengths of a similar triangle is 12cm. What is the sum of the possible perimeters of this other triangle, in cm?

21. A broken calculator only has the functions of multiplying by 2 and adding 1. If Bob starts with the number 3, what is the least number of operations he must perform to reach 101?

22. Let the function $f(x)$ have the property $kf(x) = f(kx)$. If $f(12) = 108$, compute $f(23)$.

23. A lucky number is a number that has exactly one pair of repeated digits. For example, 232 and 100 are lucky numbers while 1010 and 222 are not. How three-digit multiples of five are lucky numbers?

24. A palindrome is a number that reads the same forwards and backwards, such as 1551 or 38783. How many 5-digit palindromes are divisible by 4?

25. Solve for x if $\sqrt{x + 5\sqrt{x + 5\sqrt{x + 5 \cdots}}} = 10$.

26. What is the product of the solutions to the equation $(5 + |x - 10|)^2 = 100$?

27. There are 5 cards, numbered 1, 2, 3, 4, and 5. Matt chooses three cards. If the probability that the three cards can be arranged to form a three-digit multiple of 4 is expressed as $\frac{a}{b}$ where a and b are relatively prime, what is $a + b$?

28. In $\triangle ABC$, $\overline{AB}$ has side length 4. A point D is constructed on $\overline{BC}$, such that $CD = 6$. $\overline{AC}$ is then extended until a point E, such that $\overline{AD} = \overline{DE}$, and $\angle DEC \cong \angle BAD$. What is the product of $\overline{AC}$ and $\overline{BD}$?

29. Square $ABCD$ has side length 1. Point E is chosen on the outside of the square so that ABE is an isosceles right triangle with hypotenuse AB. If DE can be written in the form $\sqrt{\frac{a}{b}}$, where $\gcd(a, b) = 1$, find $a + b$.

30. Woody makes marks on a 120 inch tree log at points where they divide the log into 30 pieces, then makes marks at points that divide the log into 20 pieces, and finally marks points that divide the log into 12 pieces. If Woody cuts the tree log at all the points where there is a mark, how many pieces of the log does he end up with?

31. Fibonacci numbers are defined as $F_0 = 0, F_1 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for $n > 1$. What is the number of ways for a subset of any three distinct numbers chosen from $\{F_1, F_2, F_3, \cdots, F_{15}\}$ to multiply to a multiple of 10?

32. At his national karate competition, Nikhil must win against 3 opponents in a row in order to be *satisfied.* Every match, Nikhil has a $\frac{1}{3}$ chance of winning. The probability that Nikhil becomes satisfied if he is scheduled for 5 matches is $\frac{m}{n}$ for relatively prime integers m and n. What is $m + n$?

33. Thomas and Jerome are playing a game where they flip a fair coin repeatedly. If the coin lands on heads 3 times (not necessarily consecutively), the game ends and Thomas wins. If the coin lands on tails 4 times (also not necessarily consecutively), the game ends and Jerome wins. The probability that Jerome wins the game is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

34. Let n be the number of ordered triplets (x_1, x_2, x_3) such that each x_i is nonnegative and even, and $x_1 + x_2 + x_3 = 56$. Compute n.

35. How many distinct integers a, b, c exist such that $1 \leq a, b, c \leq 10$ and $\gcd(a, b, c) = 1$?

36. Autumn, Erez, Alicia, Yul, and David are dividing 13 pieces of candy among themselves. Yul is greedy and needs at least 5 pieces to be happy, while Alicia needs 2 pieces or less to be happy. Everyone else is happy, regardless of how many pieces of candy they get. How many ways can the candy be distributed so that everyone is happy?

37. Find the area of the region that is inside the equation $x^2 \leq (2 - y)(2 + y)$ and outside $|x| + |y| \leq 2$. The answer is of the form $a\pi + b$. Calculate $a + b$.

38. Compute the remainder when $1 + 3 + 3^2 + 3^3 + \cdots 3^{200}$ is divided by 7.

39. Let ABC be a triangle with $BC < CA < AB$, and $CA = 10$. How many possible integer values are there for the area of ABC?

40. How many 11-letter strings are there such that five letters spell "lance" in the order they appear and the other six spell "nikhil", also in the order they appear?