Joe Holbrook Memorial Math Competition

7th Grade

October 17, 2021

General Rules

- You will have **75 minutes** to solve **40 questions**. Your score is the number of correct answers.
- Only answers recorded on the answer sheet will be graded.
- This is an individual test. Anyone caught communicating with another student will be removed from the exam and their score will be disqualified.
- Scores will be posted on the website. Please do not forget your ID number, as that will be the sole means of identification for the scores.
- You may use the following aids:
  - Pencil or other writing utensil
  - Eraser
  - Blank scrap paper
- You may not use the following aids:
  - The Internet
  - Books or other written sources
  - Other people
  - Calculator or other computing device
  - Compass
  - Protractor
  - Ruler or straightedge

Other Notes

- Please input your answers into the Google form provided by your proctor.
- All answers are integers. Make sure you do not make any typing mistakes, as you will not be given credit if you do so.
- You do not need to write units in your answers.
- Ties will be broken by the number of correct responses to questions 31 through 40. Further ties will be broken by the number of correct responses in the last five questions.

1. Alice and Bob evaluate the expression $9 + 6 \div 3$. Alice correctly follows order of operations, while Bob calculates the expression from left to right. What is the positive difference of their answers?

2. Galadriel is reading a book. On Monday she reads $\frac{1}{4}$ of the book, on Tuesday she reads $\frac{1}{3}$ of the book, and on Wednesday she reads the last 55 pages of the book. How many pages are there in the book?

3. Bob wants to nicely wrap a gift for his friend, Bobette. He has 4 colors of wrapping paper, 5 different boxes, and 2 types of ribbons. How many ways can Bob wrap the present if he needs one wrapping paper, one box, and one ribbon?

4. Rufus, a well-trained dog, barks depending on the hour of day. Like a clock, he will bark once at 1 o'clock, twice at 2 o'clock, and so on and so forth, until he barks 12 times at 12 o'clock, at which point the cycle repeats again. Find the maximum number of times Rufus barks within a 18 hour period.

5. At a certain amusement park, there is a Ferris wheel and a roller coaster. To ride the Ferris wheel, it costs 9 dollars and to ride the roller coaster it cost 5 dollars. On a certain day, the Ferris wheel and roller coaster were ridden a total of 120 times, and the amusement park made 740 dollars. How many times was the Ferris wheel ridden on that day?

6. A palindrome is a number which is the same if read forwards or backwards. Alicia only goes to bed when the time on her phone is a palindrome. It is currently 12:21 a.m. If she doesn't go to bed now, how many minutes does she have to wait until she can go to bed? (Leading zeroes and colons are disregarded, so 02:00 a.m. would be read as the number 200.)

7. Jaiden has 27 apples. He wants to give his friend Lance more than half of them in such a way that exactly one of them will have a number of apples that's a multiple of 4, exactly one of them will have a number of apples that's a multiple of 5, and exactly one of them will have a number of apples that's a multiple of 7. How many apples does Jaiden give away?

8. What is $1 + 2 - 3 + 4 + 5 - 6 + \cdots + 97 + 98 - 99$?

9. A triangle has sides of length 6, 8, and 10 units. Its area is x square units, and the perimeter of an equilateral octagon is x units. What is the side length of this octagon?

10. The product of the digits of a ten-digit number is 105. What is the sum of these digits?

11. An anagram is a word formed by rearranging the letters of another word. For example, the word *listen* is an anagram of the word *silent*. A palindrome is a word that reads the same front and back, such as the word *pop*. How many anagrams of the word *racecar* are palindromes, including itself? (Assume that a word merely consists of a sequence of letters.)

12. Pifans like to repeatedly say the first 15 digits of π, or 3.14159265358979, at a speed of 2 digits per second. After 154 seconds, what is the sum of digits that a Pifan will have said?

13. Nikhil wants to make a viral video by rolling two fair icosahedral dice (20-sided dice) and having both land on 20. Eventually, he succeeds! To prove the dice aren't rigged, he rolls them again, hoping to get no 20s. Let the probability he doesn't roll a 20 on either die be p. If $p = \frac{n}{m}$ with n, m in lowest terms, then what is $n + m$?

14. Jaiden is throwing a party, and no party would be complete without party hats! There are three offers for party hats at the store: 25 for \$10, 10 for \$5, or 1 for \$1. Jaiden wants to get at least 38 party hats; what is the minimum amount of money he can spend, in dollars? Tax or any other additional fees are ignored.

15. There are four different senior English teachers at BCA, and each student has an equal probability of being assigned to each teacher. The probability that Autumn, David, and Erez are all assigned to the same teacher is $\frac{a}{b}$, where a and b are positive relatively prime integers. What is $a + b$?

16. Lance has a playlist with 17 songs, two of which are *Weekend* and *Good News*. If Lance plays the entire playlist once on shuffle (that is, the songs play in a random order), the probability that *Good News* plays right after *Weekend* is $\frac{a}{b}$ where a and b are positive relatively prime integers. What is $a + b$?

17. Alicia and Yul are taking a walk together. Alicia is a law-abiding citizen: from point A, she walks directly across a 15-foot street, then turns 90° to the left and walks on the sidewalk for 20 more feet to reach point B. However, Yul insists on walking in the middle of the street, so she walks directly from point A to point B. Assuming they both walk in straight lines, how many more feet does Alicia have to walk?

18. There is an equilateral triangle glued on each side of a square of length 2. The triangles are folded up to produce a pyramid. If the volume of the pyramid can be expressed as $\frac{a\sqrt{b}}{c}$, what is $a + b + c$?

19. Let $ABCD$ be a square with side length 6. The midpoint of $\overline{AB}$ is M, and E and F lie on $\overleftrightarrow{CD}$ such that $EC = CD = DF$ and the four points are distinct. What is the area common to $ABCD$ and $\triangle MEF$?

20. David is running around the school track. He is very fit, so the first mile takes him 8 minutes and 30 seconds to complete. However, every additional mile he runs takes him x seconds longer to run than the previous mile. If it takes him exactly an hour to run 6 miles, find x.

21. John Smith wants to go hunting in the woods. He brings 12 other men along with him on the adventure. John Smith decides to split his 12 men into two groups of 6 (he doesn't join either group), and the order of the groups doesn't matter. How many ways can he do this without restrictions?

22. Autumn likes bothering David. Every time she pokes David, the chance that he throws his stuffed turtle at her increases additively by $\frac{1}{10}$, beginning at $\frac{1}{10}$ the first time she pokes him. Once he throws his stuffed turtle at her, she stops poking him. The probability David throws his stuffed turtle at her the third time she pokes him is $\frac{a}{b}$, where a and b are positive relatively prime integers. What is $a + b$?

23. Allen draws a square. He then inscribes a circle within the square, inscribes another square in that circle, and finally inscribes a circle within the smaller square. The ratio of the area of the smaller circle to the area of the original square can be written in the form $\frac{a\pi}{b}$, where $\gcd(a, b) = 1$. Find $a + b$.

24. Erez loves cookies-and-cream ice cream. At his ice cream parlor, you can choose from the flavors vanilla, chocolate, strawberry, and cookies-and-cream, but he requires you to buy at least one scoop of cookies-and-cream. You can buy multiple scoops of the same flavor, and different arrangements of the same scoops are considered the same. If you buy 3 scoops of ice cream on a cone, how many possible orders can you get?

25. Farmer John ties his two cows, Bessie and Elsie, to a small shed. The base of the shed is rectangle $ABCD$, where $AB = 4$ yards and $BC = 2$ yards. John ties Bessie to corner A with a 3 yard rope and ties Elsie to corner B with a 1 yard rope. If the combined area of grass that Bessie and Elsie can graze on in square yards is $\frac{a\pi}{b}$ in lowest terms, then what is $a + b$?

26. The integer n is the smallest positive multiple of 45 such that every digit of n is either 0 or 6. What is $\frac{n}{45}$?

27. What is the sum of all integers x such that $|x^2 - 10x + 16|$ is prime?

28. Nikhil has 5 potatoes which he affectionately names Andrew, Bandrew, Candrew, Dandrew, and Endrew. However, Andrew and Bandrew don't like their names so they decide to change exactly one letter of their name into another letter. How many ways can they do this so all the potatoes still have unique names? (Notice that Andrew and Endrew have 6 letters while Bandrew, Candrew, Dandrew have 7).

29. How many ways can Michael place 5 rooks on a 6×6 chessboard so that no rooks can attack each other (that is, no two rooks are on the same row or same column)?

30. There is a geometric sequence with positive first term a_1 and ratio r. If $a_2 = \text{lcm}(a_1, r)^2$, and a_{17} is 17^{34}, what is $a + r$?

31. Dory is counting the number of fish in a coral reef, but she forgets that the digit 3 exists and skips it while counting. For example, she will count 629, then 640, skipping all the numbers in between because they include a 3. If she counts 456 fish in the reef, how many fish are actually there?

32. Dumbo is given some peanuts. Before eating them he separates them into groups of 7 and finds that he has 3 left over. He then reorganizes them into groups of 11 and finds that he has 9 left over. If Dumbo is given less than 300 peanuts, what is the maximum possible number of peanuts he was given?

33. Two ants are racing around a clock face. They both start at the 12, and begin running clockwise at the same time. One ant does 3 laps around the clock per hour, and the other does 5 laps around the clock per hour. The next time the ants are next to one another, they are closest to the number n on the clock. What is n?

34. Autumn has not been getting enough sleep. She has a total of 18 hours each day she can use for either working on her problem set or sleeping. For each hour less than 9 hours that she sleeps, her problem solving speed drops by 10% from her normal speed, and for each hour more than 9 hours that she sleeps, her speed increases by 10%. For example, if she sleeps 10 hours, she solves problems at 110% of her normal speed, and if she sleeps 7.5 hours, she solves problems at 85% of her normal speed. Let x be the number of hours that Autumn should sleep to maximize the number of problems she can solve. What is $10x$?

35. While handing out balls at your birthday party, you notice that you distributed 8 balls of different brands to three of your best friends such that each of them got at least one ball, and no two of them got the same number of balls. Find the number of ways in which you could have distributed the balls.

36. Emily moves to Octoville, where everyone uses base-8. She lives in a house numbered $abcd$, where a, b, c, d are not necessarily distinct integers and $1 \leq a, b, c, d \leq 7$. She observes that the base-10 number $abcd$ is divisible by 9. How many sequences $abcd$ are there such that the base-8 number $abcd$ is also divisible by 9?

37. There exist integers a and b such that 2^a and 3^b have remainders of 9 and 4, respectively, when divided by 71. What is the remainder when 6^{ab} is divided by 71?

38. The positive integers a, b, c, and d satisfy

$$\begin{aligned} ab + 2a + 2b &= 212 \\ bc + 2b + 2c &= 86 \\ cd + 2c + 2d &= 196. \end{aligned}$$

What is the sum of the possible values of $a + d$?

39. What is the largest integer n such that $n^3 + 50$ is a multiple of $n + 2$?

40. Let $ABCD$ be a convex quadrilateral with points M and N being the midpoints of AB and BC respectively. If $MN = 5$, $DN = 12$, $DM = 13$, and the area of triangle ADC is 27, compute the area of $ABCD$.