Joe Holbrook Memorial Invitational Competition

4th Grade

March 20, 2022

General Rules

- You will have **90 minutes** to solve **16 questions**. Your score is the number of correct answers.
- Only answers recorded on the answer sheet will be graded.
- This is an individual test. Anyone caught communicating with another student will be removed from the exam and their score will be disqualified.
- You may use the following aids:
  - Pencil or other writing utensil
  - Eraser
  - Blank scrap paper
- You may not use the following aids:
  - The Internet
  - Books or other written sources
  - Other people
  - Calculator or other computing device
  - Compass
  - Protractor
  - Ruler or straightedge

Other Notes

- Please input your answers into the Google form provided by your proctor.
- **All answers are integers.** Please enter them with no spaces in between. For example, enter -7 not - 7.
- Do not include commas in your answers. For example, the number one thousand is to be entered 1000 not 1,000.
- You do not need to write units in your answers.
- Make sure you do not make any typing mistakes, as you will not be given credit if you do so.
- Ties will be broken by the number of correct responses to the last 5 problems. Further ties will be broken by the number of correct responses to the previous 5 problems, etc.
- Keep in mind that the JHMIC is a difficult contest and very different from school assessments. If you even get a few questions right, you should feel proud of yourself!

1. A clown number is a positive integer with the following properties. All clown numbers are divisible by 3 and 5. The sum of the squares of the digits of a clown number is prime. What is the smallest clown number?

2. Every woodchuck chucks wood at a constant rate. If 6 woodchucks can chuck 2 logs in 16 hours, then how many woodchucks are needed to chuck 5 logs in 10 hours?

3. Agrabah is a city with many straight streets. In fact, there are exactly 2020 perfectly parallel streets and 1 main street that intersects all of them. If no streets are perpendicular to each other, what percent of the corners (angles between two streets) are acute? (Note: Streets are not curved)

4. The probability that a randomly chosen 4-digit number (with no leading zeroes) is divisible by 7, 11, and 13 is $\frac{a}{b}$, where a and b are positive relatively prime integers. What is $a + b$?

5. In a certain language, words are made up of the letters A-J, start and end in a consonant, and must have at least one vowel between each consonant. How many possible 5-letter words are there?

6. Moana is sailing her boat at a constant speed between two islands called island A and island B that are 21 miles apart. Since the wind is blowing toward island B (at a constant speed), it takes her 3 hours to sail from island A to island B but 4 hours to sail from island B to island A. The wind speed can be written as $\frac{m}{n}$ miles per hour, where m and n are relatively prime positive integers. Find $m + n$.

7. Erez has forgotten his password! He remembers that it is a 6-digit binary number (base 2), and that the first three digits are 111. He also remembers that when converted to base 10, the password is prime. What is the maximum possible value for the last three digits of his password (in binary)? (Exclude any leading 0s in your answer.)

8. There exists rectangle $ABCD$ with $AB = 5$ and $BC = 9$. Let X and Y be points on BC and AD, respectively. How many possible integer values are there for XY?

9. Four people, 1, 2, 3, and 4, are playing a game, where 2, 3, and 4 make up a team. First, 1 shoots at a member of the team. Then each member of the team shoots at 1, and then 1 shoots at the team again, and so on. If a person is shot, then they are out of the game and cannot shoot anyone else. A team loses if all members are out (or if its sole member is eliminated). If both teams play optimally, and player i has a $\frac{1}{i}$ chance of hitting their shot, the probability that 1 wins can be written in the form $\frac{a}{b}$ where a, b have a greatest common divisor of one. What is $a + b$?

10. David is traveling by train to see his friend Autumn. The train at a constant speed in a straight line from David to Autumn. At the start of his trip, David sends a messenger pigeon to Autumn, to tell her that he is coming. When Autumn receives the pigeon, she immediately sends it back to David. Likewise, when David receives the returning pigeon, he will immediately send it back as well. The two will repeat this process, sending the pigeon back and forth, until David finally arrives at Autumn's station. To the nearest integer, how many times faster than the train must the pigeon fly for the total distance the pigeon travelled to be at least twice that of David?

11. Jim wants to distribute his 8 apples amongst himself and his four friends. Jim is a great guy, so he wants each of his friends to get at least as many apples as him (he may also decide to give zero apples to himself). How many ways are there for him to do this?

12. What is the absolute value of the sum of all integers n such that $\frac{3n + 14}{n + 3}$ is an integer?

13. Bianca wants to decorate the regular tetrahedron that sits on her shelf. A regular tetrahedron is a triangular pyramid with congruent faces. She can paint each vertex one of five colors, but she doesn't want to paint two different corners the same color. How many different ways can she paint the tetrahedron? Two different paintings are considered the same if the tetrahedron can be rotated to turn one painting into another.

14. Ling was attempting to pitch a tent on a mountain. Due to the slope of the mountain, the open side of the tent formed a triangle $\triangle ABC$, where $AB = 9$, $AC = 11$, and $BC = 10$. He ties a string from point A to the midpoint of $\overline{BC}$. Find the square of the length of the string.

15. If $xy = 3$ and $x + y = 5$, what is $x^4y + y + x + xy^4$?

16. Abby and Betty are playing a game. Every turn, Abby flips a coin, and Betty randomly draws a card without replacement from a standard deck of 52 cards. If Abby gets heads before Betty draws the ace of spaces, she wins the game; if not, Betty wins. (This means that if Abby gets heads on the same turn that Betty draws the ace of spades, Betty wins). Let P be the probability that Betty wins. What is the closest integer to $\frac{1}{P}$?