Joe Holbrook Memorial Invitational Competition

5th Grade

March 20, 2022

General Rules

- You will have **90 minutes** to solve **16 questions**. Your score is the number of correct answers.
- Only answers recorded on the answer sheet will be graded.
- This is an individual test. Anyone caught communicating with another student will be removed from the exam and their score will be disqualified.
- You may use the following aids:
  - Pencil or other writing utensil
  - Eraser
  - Blank scrap paper
- You may not use the following aids:
  - The Internet
  - Books or other written sources
  - Other people
  - Calculator or other computing device
  - Compass
  - Protractor
  - Ruler or straightedge

Other Notes

- Please input your answers into the Google form provided by your proctor.
- **All answers are integers.** Please enter them with no spaces in between. For example, enter -7 not - 7.
- Do not include commas in your answers. For example, the number one thousand is to be entered 1000 not 1,000.
- You do not need to write units in your answers.
- Make sure you do not make any typing mistakes, as you will not be given credit if you do so.
- Ties will be broken by the number of correct responses to the last 5 problems. Further ties will be broken by the number of correct responses to the previous 5 problems, etc.
- Keep in mind that the JHMIC is a difficult contest and very different from school assessments. If you even get a few questions right, you should feel proud of yourself!

1. Train A leaves from the station every 30 minutes, and Train B leaves from the station every 18 minutes. Train A and Train B leave the station together at 1:00 PM. If Tommy and Timmy arrive at the train station at 1:30 PM, how many minutes must they wait for the trains to leave the station at the same time again?

2. Find the smallest positive integer $x > 1$ such that the smallest prime factor of x is greater than the smallest prime factor of $x + 6$.

3. Belle was organizing the books in her library. She found that $\frac{1}{16}$ of her books were science fiction, $\frac{2}{7}$ were memoirs, $\frac{1}{2}$ were mystery, $\frac{1}{8}$ were reference books and the rest were on miscellaneous topics. If she has a total of 224 books, how many ways are there to arrange the miscellaneous books in a line?

4. In a certain language, words are made up of the letters A-J, start and end in a consonant, and must have at least one vowel between each consonant. How many possible 5-letter words are there?

5. Moana is sailing her boat at a constant speed between two islands called island A and island B that are 21 miles apart. Since the wind is blowing toward island B (at a constant speed), it takes her 3 hours to sail from island A to island B but 4 hours to sail from island B to island A. The wind speed can be written as $\frac{m}{n}$ miles per hour, where m and n are relatively prime positive integers. Find $m + n$.

6. Merida is practicing her archery. The board she is trying to hit is in the shape of a circle split into sections. It consists of 8 evenly spaced concentric rings that alternate between black and white (like the typical target pattern). The radii of the circles are 1,2,3,4,5,6,7, and 8 inches. If the probability that she will hit two different colors with two arrows in simplified form is $\frac{a}{b}$, what is $a + b$?

7. Erez has forgotten his password! He remembers that it is a 6-digit binary number (base 2), and that the first three digits are 111. He also remembers that when converted to base 10, the password is prime. What is the maximum possible value for the last three digits of his password (in binary)? (Exclude any leading 0s in your answer.)

8. There exists rectangle $ABCD$ with $AB = 5$ and $BC = 9$. Let X and Y be points on BC and AD, respectively. How many possible integer values are there for XY?

9. Four people, 1, 2, 3, and 4, are playing a game, where 2, 3, and 4 make up a team. First, 1 shoots at a member of the team. Then each member of the team shoots at 1, and then 1 shoots at the team again, and so on. If a person is shot, then they are out of the game and cannot shoot anyone else. A team loses if all members are out (or if its sole member is eliminated). If both teams play optimally, and player i has a $\frac{1}{i}$ chance of hitting their shot, the probability that 1 wins can be written in the form $\frac{a}{b}$ where a, b have a greatest common divisor of one. What is $a + b$?

10. Two circles O_1 and O_2 of radius r are externally tangent to each other and internally tangent to a circle O_3 of radius $2r$. In addition, circle O_4 of radius 1 is externally tangent to O_1 and O_2 and internally tangent to O_3. The area of the circle with radius r can be expressed as $\frac{a}{b}\pi$, where a and b are relatively prime positive integers. Find $a + b$.

11. Two divisors a, b of $8!$, not necessarily distinct, are chosen independently at random. The probability that a divides b can be expressed in the form $\frac{m}{n}$, for relatively prime positive integers m and n. Compute $m + n$.

12. Marj and Nick decide to go swimming in a perfectly circular lake. Marj can swim at 4 miles per hour while Nick can swim at 3 miles per hour. They start at different points on the circumference of the lake and swim in a straight line, stopping when they reach the shore. They only cross paths once. When they cross paths, Marj has been swimming for 25 minutes and Nick has been swimming for 10 minutes. If Marj ends up swimming $\frac{3}{4}$ of a mile more than Nick, the distance Marj swam can be represented as the simplified fraction $\frac{a}{b}$. What is $a + b$?

13. David's calculator broke! All the buttons for even digits function properly, but the buttons for the odd digits are completely nonfunctional. Luckily, he has a plan. If he wants to input an integer, like 2479, which would require him to use a broken button, he will instead input the closest possible integer containing only even digits. For example, instead of 2479, David would input 2480. Similarly, instead of 357, David

would input 400. In the event that two alternative inputs are equally good approximations, David will input either one of them. David inputted 2000 + 600 + 40 + 8 into his calculator. He knows that the displayed result will be at most k away from his intended result. What is k?

14. How many ways are there to climb a flight of stairs that has 9 steps if you can only climb 1, 2, or 3 steps at a time?

15. The sequence $\{a_n\}$ only has positive terms. If the sum of the terms from the first term to the nth term is represented by S_n, then

$$S_{n+1} + S_n = a_{n+1}^2.$$

If $a_1 = 6$, what is the value of a_{20}?

16. Let $f(x) = \lfloor x^2 \rfloor + (\lfloor x \rfloor)^2$ where $\lfloor n \rfloor$ represents the greatest integer less than or equal to n. Find the 50th smallest value of k such that $f(x) = k$ has a solution for a positive real number x.