Joe Holbrook Memorial Invitational Competition

6th Grade

March 20, 2022

General Rules

- You will have **90 minutes** to solve **16 questions**. Your score is the number of correct answers.
- Only answers recorded on the answer sheet will be graded.
- This is an individual test. Anyone caught communicating with another student will be removed from the exam and their score will be disqualified.
- You may use the following aids:
  - Pencil or other writing utensil
  - Eraser
  - Blank scrap paper
- You may not use the following aids:
  - The Internet
  - Books or other written sources
  - Other people
  - Calculator or other computing device
  - Compass
  - Protractor
  - Ruler or straightedge

Other Notes

- Please input your answers into the Google form provided by your proctor.
- **All answers are integers.** Please enter them with no spaces in between. For example, enter -7 not - 7.
- Do not include commas in your answers. For example, the number one thousand is to be entered 1000 not 1,000.
- You do not need to write units in your answers.
- Make sure you do not make any typing mistakes, as you will not be given credit if you do so.
- Ties will be broken by the number of correct responses to the last 5 problems. Further ties will be broken by the number of correct responses to the previous 5 problems, etc.
- Keep in mind that the JHMIC is a difficult contest and very different from school assessments. If you even get a few questions right, you should feel proud of yourself!

1. Find the smallest positive integer $x > 1$ such that the smallest prime factor of x is greater than the smallest prime factor of $x + 6$.

2. Erez and Alicia want to make an origami stellated icosahedron, which requires 30 origami sonobe units. If Erez takes 10 seconds to fold one origami sonobe and Alicia takes 15 seconds to fold one origami sonobe, then how long, in minutes, does it take the two of them working together to make one stellated icosahedron?

3. The probability that a randomly chosen 4-digit number (with no leading zeroes) is divisible by 7, 11, and 13 is $\frac{a}{b}$, where a and b are positive relatively prime integers. What is $a + b$?

4. In trapezoid $ABCD$, $AB \parallel CD$, $AB \perp BC$, $AB = 12$, $BC = 15$, and $CD = 20$. What is the length of AD?

5. A positive integer is called a boom number if its digits are strictly increasing and its last digit is 9 (for example, 9 and 2359 are boom numbers, but 219 and 559 are not). How many boom numbers are there less than 1000?

6. Let a, b be positive integers. If $(a + b)(a - b) = 2021$, find the sum of all possible values of ab.

7. There exists rectangle $ABCD$ with $AB = 5$ and $BC = 9$. Let X and Y be points on BC and AD, respectively. How many possible integer values are there for XY?

8. David is traveling by train to see his friend Autumn. The train at a constant speed in a straight line from David to Autumn. At the start of his trip, David sends a messenger pigeon to Autumn, to tell her that he is coming. When Autumn receives the pigeon, she immediately sends it back to David. Likewise, when David receives the returning pigeon, he will immediately send it back as well. The two will repeat this process, sending the pigeon back and forth, until David finally arrives at Autumn's station. To the nearest integer, how many times faster than the train must the pigeon fly for the total distance the pigeon travelled to be at least twice that of David?

9. Jaiden the corn farmer and Nikhil the potato farmer are having a land dispute and decide to settle it with a sequence of fair coin flips. Out of 5 flips, if there are 3 or more heads, Jaiden wins, and Nikhil wins otherwise. However, Jaiden rigs the coin so that it now has a $\frac{3}{4}$ chance of landing heads. By rigging the coin, Nikhil's chances of winning have decreased by $\frac{m}{n}$ for relatively prime positive integers m and n. What is $m + n$?

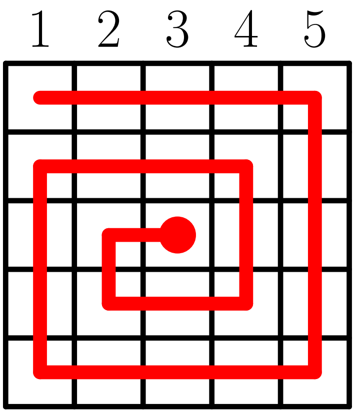
10. Marj and Nick decide to go swimming in a perfectly circular lake. Marj can swim at 4 miles per hour while Nick can swim at 3 miles per hour. They start at different points on the circumference of the lake and swim in a straight line, stopping when they reach the shore. They only cross paths once. When they cross paths, Marj has been swimming for 25 minutes and Nick has been swimming for 10 minutes. If Marj ends up swimming $\frac{3}{4}$ of a mile more than Nick, the distance Marj swam can be represented as the simplified fraction $\frac{a}{b}$. What is $a + b$?

11. Two divisors a, b of $8!$, not necessarily distinct, are chosen independently at random. The probability that a divides b can be expressed in the form $\frac{m}{n}$, for relatively prime positive integers m and n. Compute $m + n$.

12. Find the smallest positive integer n such that n^3 has 7 times as many divisors as n.

13. What is the smallest integer greater than 2022_{10} that is a palindrome in both octal (base 8) and hexadecimal (base 16)? Express your answer in base 10.

14. Let there be a 5 by 5 grid and 5 rocks, labeled 1-5. Each rock can only be placed in its corresponding column, and no two rocks can be placed in the same row. We then take this path, starting in the upper left hand corner, picking up rocks in the order we find them. How many permutations of rocks can we end up with?



15. Let $p_1, p_2, \ldots, p_6$ be an increasing arithmetic sequence of primes. Find the smallest possible value of p_6.

16. Let $ABCD$ be a trapezoid with $AB \parallel CD$. The two rays from point D to the midpoint of AB and to the midpoint of BC trisect $\angle ADC$. Given that $AB = 108$ and $CD = 42$, the area of $ABCD$ can be expressed as $m\sqrt{n}$ for positive integers m and n, with n squarefree. Compute $m + n$.