Joe Holbrook Memorial Invitational Competition

8th Grade

March 20, 2022

General Rules

- You will have **90 minutes** to solve **16 questions**. Your score is the number of correct answers.
- Only answers recorded on the answer sheet will be graded.
- This is an individual test. Anyone caught communicating with another student will be removed from the exam and their score will be disqualified.
- You may use the following aids:
  - Pencil or other writing utensil
  - Eraser
  - Blank scrap paper
- You may not use the following aids:
  - The Internet
  - Books or other written sources
  - Other people
  - Calculator or other computing device
  - Compass
  - Protractor
  - Ruler or straightedge

Other Notes

- Please input your answers into the Google form provided by your proctor.
- **All answers are integers.** Please enter them with no spaces in between. For example, enter -7 not - 7.
- Do not include commas in your answers. For example, the number one thousand is to be entered 1000 not 1,000.
- You do not need to write units in your answers.
- Make sure you do not make any typing mistakes, as you will not be given credit if you do so.
- Ties will be broken by the number of correct responses to the last 5 problems. Further ties will be broken by the number of correct responses to the previous 5 problems, etc.
- Keep in mind that the JHMIC is a difficult contest and very different from school assessments. If you even get a few questions right, you should feel proud of yourself!

1. Agrabah is a city with many straight streets. In fact, there are exactly 2020 perfectly parallel streets and 1 main street that intersects all of them. If no streets are perpendicular to each other, what percent of the corners (angles between two streets) are acute? (Note: Streets are not curved)

2. In a certain language, words are made up of the letters A-J, start and end in a consonant, and must have at least one vowel between each consonant. How many possible 5-letter words are there?

3. Moana is sailing her boat at a constant speed between two islands called island A and island B that are 21 miles apart. Since the wind is blowing toward island B (at a constant speed), it takes her 3 hours to sail from island A to island B but 4 hours to sail from island B to island A. The wind speed can be written as $\frac{m}{n}$ miles per hour, where m and n are relatively prime positive integers. Find $m+n$.

4. Merida is practicing her archery. The board she is trying to hit is in the shape of a circle split into sections. It consists of 8 evenly spaced concentric rings that alternate between black and white (like the typical target pattern). The radii of the circles are 1,2,3,4,5,6,7, and 8 inches. If the probability that she will hit two different colors with two arrows in simplified form is $\frac{a}{b}$, what is $a+b$?

5. At 7:00 AM, Aminah's alarm buzzes. Whenever Aminah hears her alarm, she has a 5% chance of getting up from bed and a 95% chance of hitting the snooze button, in which case the alarm resumes buzzing exactly one minute later. The expected time that Aminah gets up is n minutes after 7:00. Find n.

6. At midnight, the hour and minute hands of an analog clock lie one on top of the other. The third time after midnight when the hands of this clock do the same is closest to h hours and m minutes after midnight. Find $100h+m$.

7. The polynomial $x^3-17x^2-105x+49$ has roots r, s, and t. The expression
$$\left|\frac{r-s}{st}+\frac{s-t}{rt}+\frac{t-r}{rs}\right|$$
can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

8. Marj and Nick decide to go swimming in a perfectly circular lake. Marj can swim at 4 miles per hour while Nick can swim at 3 miles per hour. They start at different points on the circumference of the lake and swim in a straight line, stopping when they reach the shore. They only cross paths once. When they cross paths, Marj has been swimming for 25 minutes and Nick has been swimming for 10 minutes. If Marj ends up swimming $\frac{3}{4}$ of a mile more than Nick, the distance Marj swam can be represented as the simplified fraction $\frac{a}{b}$. What is $a+b$?

9. Find the smallest positive integer n such that n^3 has 7 times as many divisors as n.

10. Ling was attempting to pitch a tent on a mountain. Due to the slope of the mountain, the open side of the tent formed a triangle $\triangle ABC$, where $AB=9$, $AC=11$, and $BC=10$. He ties a string from point A to the midpoint of $\overline{BC}$. Find the square of the length of the string.

11. Bianca wants to decorate the regular tetrahedron that ominously sits on her shelf. A regular tetrahedron is a triangular pyramid with congruent faces. She can paint each vertex one of five colors, but she doesn't want to paint two different corners the same color. How many different ways can she paint the tetrahedron? Two different paintings are considered the same if the tetrahedron can be rotated to turn one painting into another.

12. A positive integer n is denoted as *touchy-feely* if (225^n+227^n) is an integer multiple of 791. Find the number of touchy-feely numbers between 1 and 2022 inclusive.

13. Let Ω be a semicircle with diameter $AB=10$. There exists points C and D on Ω such that AC and BD intersect at point E inside the semicircle, with $AE=6$ and $CE=2$. If AD^2 can be written as $\frac{m}{n}$ for relatively prime positive integers m and n, compute $m+n$.

14. Jim goes to the store and buys so many apples, that even after breakfast, he still has 16 left! He wants to split these 16 indistinguishable apples amongst his 6 friends: Abuna, Balthazzar, Camillus, Dakari, Eido, and Fen, such that the first three each get a number of apples that is a multiple of 3 (possibly 0), and the last three each get a number of apples that is not a multiple of 3. How many ways are there for Jim to distribute his apples?

15. Let a and b be two positive integers, neither of which divides the other, such that $\text{lcm}(a, b) = \gcd(a, b)^{\gcd(a,b)}$. Compute the smallest possible value of $a + b$.

16. Dr. Jean-Marie has two zeroes written on the board. Every minute, he tells one of his students to go up to the board, pick one of the two numbers randomly, and increase it by 1. During this process, he also keeps track of what the minimum number on the board is, and what the maximum number is. During the first 10 minutes, let A be the expected number of times the value of the minimum number on the board changes, and let B be the expected number of times the value of the maximum number on the board changes. For example, if the two numbers are 2 and 6 at some point, and the next minute the 2 is increased to 3, then the minimum has changed while the maximum has not. If $|B - A|$ can be written as $\frac{m}{n}$, for relatively prime positive integers m and n, find $m + n$.