

Answer Sheet

Name: _____

ID: _____

1.	11.	21.	31.
2.	12.	22.	32.
3.	13.	23.	33.
4.	14.	24.	34.
5.	15.	25.	35.
6.	16.	26.	36.
7.	17.	27.	37.
8.	18.	28.	38.
9.	19.	29.	39.
10.	20.	30.	40.

FOR GRADER USE ONLY:

Score 1	Score 2	Score 3	Score 4
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Total Score:

Joe Holbrook Memorial Math Competition

4th Grade

October 16, 2022

General Rules

- You will have **75 minutes** to solve **40 questions**. Your score is the number of correct answers.
- Only answers recorded on the answer sheet will be graded.
- This is an individual test. Anyone caught communicating with another student will be removed from the exam and their score will be disqualified.
- Scores will be posted on the website. Please do not forget your ID number, as that will be the sole means of identification for the scores.
- You may use the following aids:
 - Pencil or other writing utensil
 - Eraser
 - Blank scrap paper
- You may not use the following aids:
 - The Internet
 - Books or other written sources
 - Other people
 - Calculator or other computing device
 - Compass
 - Protractor
 - Ruler or straightedge

Other Notes

- All answers are integers. Make sure you do not make any mistakes when writing your answers, as you will not be given credit if you do so.
- You do not need to write units in your answers.
- Ties will be broken by the number of correct responses to questions 31 through 40. Further ties will be broken by the number of correct responses in the last five questions.

1. Compute the average of the first five positive integers.
2. Oh no! Lance forgot about Eshaan's birthday! As an apology, Lance buys Eshaan a concert ticket; the ticket costs \$115. If Lance has \$240 in his bank account, how much money (in dollars) will he have after he buys the ticket?
3. What is 20×22 ?
4. Find $2^{(2+0+2+2)} + (2 + 0 + 2 + 2)^2$.
5. Joh N. teaches 10 classes a day but hates 3 of them (he likes the other ones). What percent of his classes does he like? (If your answer is $a\%$, submit a).
6. Bobert has 3 dozens of eggs. How many eggs does he have? There are 12 eggs in a dozen.
7. Timmy makes 10 paper cutouts every hour and Shawn makes 50% more paper cutouts every hour than Timmy. How many more hours than Shawn does Timmy have to work, if they both make 1200 cutouts?
8. Lance can deadlift 5 pounds and Jaiden can deadlift 50 pounds. If Nikhil can deadlift twice as much as Jaiden, how many more pounds can Nikhil deadlift than Lance?
9. Lance has written 12 problems, but now he has to write the solutions to them! Each problem is rated an integer difficulty from 1 to 10, and a problem of difficulty i takes him i minutes to write a solution for. If his problems' average difficulty is 6, how many minutes will it take him to write all of his solutions?
10. Saria and her friend ordered a 15-slice pizza for lunch. Saria ate one slice and split another with her friend. What percentage of the pizza did Saria eat?
11. Eshaan listens to 14 songs a day. How many whole days will it take for him to listen to at least 293 songs?
12. Let $a \bullet b = a + b + ab$. What is $17 \bullet (1 \bullet 1)$?
13. Rose has a 20% off coupon and a \$10 off coupon for a certain clothing store. She wants to apply both of these coupons (which the store allows) for a \$100 pair of heels. What is the minimum amount Rose can pay for these heels?
14. Find the smallest odd positive prime p such that $p^2 + 4$ is not prime.
15. A square has an area of 196 inches². If a regular nonagon (nine-sided polygon) has three times the side length of the square, what is the perimeter of the nonagon (in inches)?
16. Phoenix has a 13" \times 4" sheet of chocolate cake which she wants to cut into 2" \times 2" pieces. At most how many pieces can Phoenix get?
17. King Henry VIII challenges his court to guess his favorite natural number. A wise nobleman is able to correctly determine his favorite natural number after asking three questions, all of which the king answers correctly: is it 1-digit, is it prime, and is it odd? What is the king's favorite natural number?
18. Pablo is the world's fastest jockey. He is running a race where he rides the first third (by distance) of the race at 60 mph (miles per hour), the second third of the race at 30 mph, and the final third of the race at 55 mph. What was Pablo's average speed during the race (in mph)?
19. Tasha wants to put up a picture in her living room. Her picture frame is 2 inches thick on all sides and has an outer perimeter of 66 inches. If the length of the picture frame is 2 times longer than its width, what is the length of Tasha's picture?
20. Jacob has created a language with 252 different syllables. Each syllable consists of one consonant followed by a vowel. If there are four more consonants than vowels, how many vowels are there in Jacob's alphabet?
21. Let a be a positive integer such that there are 9 positive multiples of a less than 100. Let b be a positive integer such that there are 19 positive multiples of b less than 100. How many positive multiples of ab are less than 100?
22. Justin wants to test if his new computer is working. He selects an arbitrary real number k from 0 to 10 inclusive, and asks his computer to find the root to the equation $3x + k = 0$. The probability that the output of the computer is less than $-\frac{2}{5}$ can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Compute $m + n$.

23. The road between City A and City B is 450 miles long, and cars travel at 60 mph (miles per hour) on the road. Alternatively, there is a flight from City A to City B that travels at 240 mph along a more direct 400 mile route, but it takes two hours to get to the airport and board the flight. By how many minutes is flying faster than driving?
24. A positive integer is called *fishy* if all of its digits are composite, but it is prime. Find the sum of all fishy numbers less than 100.
25. Tyrone is a master at making pizza. He is currently making an 18-inch diameter pizza with 20 slices of 2-inch diameter pepperoni. The crust is exactly 2-inches thick all around the perimeter of the pizza. Tyrone wants to cover all the dough that is not part of the crust and is not covered by pepperoni with cheese. If Tyrone needs $a\pi$ square inches of cheese, what is a ? The pizza can be assumed to be perfectly 2D.
26. Point A is located at $(2, 4)$ and point B is located at $(8, 12)$. Jack walks from A to B in a straight line. Jane walks from A to B in a semicircular path. The absolute difference between the distances they traveled can be expressed as $a\pi - b$, where a and b are positive integers. Compute $a + b$.
27. What is the units digit of $7^{2022} + 2^{2022}$?
28. The sum of all but one angle of a convex polygon (a polygon with no angle over 180 degrees) is 2022 degrees. What is the measure of the missing angle in degrees?
29. Suppose the median of the set $\{1, 2, 5, 5, 8, 9, 12, 15, x\}$ is one less than its mean. What is the sum of the possible values of x ?
30. Austin is a robot repair man. He needs a code to fix a robot, but he has forgotten the code. He knows that the first two digits of the code are 2 and 7 in that order and that it is five digits long. He also remembers that he made the code divisible by 1, 3, 4, 5, 6, 8, and 9. How many possible codes are there that satisfy the previous conditions?
31. Let $s(n)$ be the sum of digits of n . What is the smallest positive integer n such that $s(s(n)) = 11$?
32. Rectangle $ABCD$ with $AB = 6$ and $BC = 7$ is inscribed in a semicircle such that CD lies on the diameter. If the area of the semicircle is $a\pi$, find a .
33. Kathy is randomly sewing 6 buttons onto her shirt in a vertical line. She has three colors of buttons, red, orange, and yellow, and two of each. The probability that there will be at least one consecutive ordering of red, orange, yellow, on her shirt from top to bottom, can be expressed as $\frac{m}{n}$, where m, n are relatively prime positive integers. Find $m - n$.
34. Even-Steven's arch-nemesis, Odd-Todd, rigged a coin so that it would become heads 75% of the time. The two play a game together where they flip a coin 5 times. Odd-Todd is so overly confident he said that he would only win the game if he flipped 3 heads or 5 heads, as he loves odd numbers. The positive difference between the probabilities of either of them winning can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?
35. Find the sum of all positive integers N such that the second largest factor of N is 15 times its second smallest factor.
36. Billy Bobby has just entered kindergarten, and has trouble telling the difference between 6 and 9. Whenever he writes either, he has a $\frac{2}{3}$ chance of writing it correctly. The probability that he writes the number 9669 with only one wrong digit can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?
37. Two unit squares are selected in an 8×8 grid. The probability that they are horizontally or vertically adjacent can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?
38. Prabav (a goat Lover) ties his favorite goat with a 3 meter rope to the corner of a building in the shape of an equilateral triangle with a length of 2 meters. While Prabav was not looking, the goat ate as much grass as he possibly could. The total amount that the goat grazed can be expressed as $\frac{m\pi}{n}$ where m and n are relatively prime positive integers. What is $m + n$?

39. Amir is at $(0, 0)$ in the coordinate plane and is walking towards his house, which is at $(8, 8)$, such that all of his steps are either up or right and of length 1. Unfortunately, construction is being done at every point (a, b) where both a and b are odd, so Amir cannot travel to any of these points along his path. How many different routes can Amir take?
40. Two circles of radius 6 have centers O_1 and O_2 such that $O_1O_2 = 6\sqrt{3}$. If the area of the region contained in both circles can be expressed as $a\pi + b\sqrt{c}$ for integers a, b , and c , with c nonnegative and squarefree, find $a + b + c$.