

Answer Sheet

Name: _____

ID: _____

1.	11.	21.	31.
2.	12.	22.	32.
3.	13.	23.	33.
4.	14.	24.	34.
5.	15.	25.	35.
6.	16.	26.	36.
7.	17.	27.	37.
8.	18.	28.	38.
9.	19.	29.	39.
10.	20.	30.	40.

FOR GRADER USE ONLY:

Score 1	Score 2	Score 3	Score 4
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Total Score:

Joe Holbrook Memorial Math Competition

5th Grade

October 16, 2022

General Rules

- You will have **75 minutes** to solve **40 questions**. Your score is the number of correct answers.
- Only answers recorded on the answer sheet will be graded.
- This is an individual test. Anyone caught communicating with another student will be removed from the exam and their score will be disqualified.
- Scores will be posted on the website. Please do not forget your ID number, as that will be the sole means of identification for the scores.
- You may use the following aids:
 - Pencil or other writing utensil
 - Eraser
 - Blank scrap paper
- You may not use the following aids:
 - The Internet
 - Books or other written sources
 - Other people
 - Calculator or other computing device
 - Compass
 - Protractor
 - Ruler or straightedge

Other Notes

- All answers are integers. Make sure you do not make any mistakes when writing your answers, as you will not be given credit if you do so.
- You do not need to write units in your answers.
- Ties will be broken by the number of correct responses to questions 31 through 40. Further ties will be broken by the number of correct responses in the last five questions.

1. If 24% of x equals 30% of y , what percent of x is y ?
2. Tasha is a superhero with super speed. There are 7 different places all around the city that are experiencing a crime. How many different paths can Tasha take to fight all of the crimes?
3. Timmy makes 10 paper cutouts every hour and Shawn makes 50% more paper cutouts every hour than Timmy. How many more hours than Shawn does Timmy have to work, if they both make 1200 cutouts?
4. Tasha loves collecting sea shells. One day she finds 5 seashells. Every day after that (starting on the second day) she makes it a goal to collect 4 more seashells than the previous day. After 10 days, how many seashells does Tasha have in total?
5. Eshaan listens to 14 songs a day. How many whole days will it take for him to listen to 293 songs?
6. Tasha wants to put up a picture in her living room. Her picture frame is 2 inches thick on all sides and has an outer perimeter of 66 inches. If the length of the picture frame is 2 times longer than its width, what is the length of Tasha's picture?
7. Lance has written 12 problems, but now he has to write the solutions to them! Each problem is rated an integer difficulty from 1 to 10, and a problem of difficulty i takes him i minutes to write a solution for. If his problems' average difficulty is 6, how many minutes will it take him to write all of his solutions?
8. Jacob has created a language with 252 different syllables. Each syllable consists of one consonant followed by a vowel. If there are four more consonants than vowels, how many vowels are there in Jacob's alphabet?
9. Rose has a 20% off coupon and a \$10 off coupon for a certain clothing store. She wants to apply both of these coupons (which the store allows) to a \$100 pair of heels. What is the minimum amount Rose can pay for these heels?
10. Find the smallest odd positive prime p such that $p^2 + 4$ is not prime.
11. There is a $\frac{2}{5}$ chance that it will rain today. If it rains today, there is a $\frac{1}{4}$ chance that it will also rain tomorrow. If it does not rain today, there is a $\frac{3}{4}$ chance that it will rain tomorrow. The chance that it will rain tomorrow can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?
12. Let a be a positive integer such that there are 9 positive multiples of a less than 100. Let b be a positive integer such that there are 19 positive multiples of b less than 100. How many positive multiples of ab are less than 100?
13. I have 4 cupcakes. Every day starting from day 2, I double the amount of cupcakes I have and then eat three. On the first day I start with 4 cupcakes, then on the second I have 5, on the third I have 7, and so on. How many cupcakes will I have on the tenth day?
14. The road between City A and City B is 450 miles long, and cars travel at 60 mph (miles per hour) on the road. Alternatively, there is a flight from City A to City B that travels at 240 mph along a more direct 400 mile route, but it takes two hours to get to the airport and board the flight. By how many minutes is flying faster than driving?
15. What is the units digit of $2^{2022} + 5^{2022}$?
16. The city of Hackensack has decided to open a zoo with only snakes, chickens, and parrots. When Ann visited the zoo, she counted 24 heads and 38 legs. If chickens and parrots both have two legs, while snakes have none, how many snakes are there at the Hackensack Zoo?
17. The sum of all but one angle of a convex polygon (a polygon with no angle over 180 degrees) is 2022 degrees. What is the measure of the missing angle in degrees?
18. A positive integer is called *fishy* if all of its digits are composite, but it is prime. Find the sum of all fishy numbers less than 100.
19. Austin, Pablo, Tyrone, and Tasha are at the edge of the circular crater of a volcano. The line connecting Austin and Pablo is parallel to the line connecting Tyrone and Tasha. From Pablo's view, Tyrone and Austin are 40 degrees apart. This is also true from Tasha's view. What is the arc measure, in degrees, between Austin and Pablo?

20. Cathy bought 5 candy bars for 50 cents each with 16 coins. If it is known that she paid only with dimes and quarters, how many quarters did she spend?
21. Merlin is performing a party trick in which he pulls three cards out (one at a time) from a standard deck of 52 playing cards. The probability that Merlin ends up with at least two cards of the same suit can be represented as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?
22. How many calendar dates in 2022 have a prime day of the month? For example, tomorrow is October 17th, and 17 is prime.
23. A woman is standing on a field of corn wearing a circular sombrero of radius 4 feet. Unfortunately, the hat blocks the sunlight so effectively that the corn directly under it dies instantly. If the woman walks along the circumference of a circle of radius 6 feet, the area of dead corn can be written as $a\pi$. What is a ?
24. $16^x - 4^{x+\frac{3}{2}} = 128$ has an integer solution q . What is q ?
25. Andrea and Anthony are running together to be on a committee chosen randomly from 100 people, with only a president and vice president. Label any scenario in which Andrea and Anthony are vice-president and president, irrespective of which one of them gets which position *ideal*. The probability that the committee ends up being *ideal* can be written $\frac{m}{n}$, where m, n are relatively prime positive integers. Find $m + n$.
26. A triangle ABC has point D lying on \overline{AB} and point E lying on \overline{AC} . If \overline{DE} is parallel to \overline{BC} , the area of triangle $ABC = 50$ units², and $\frac{\overline{AB}}{\overline{DB}} = 5$, what is the area of triangle ADE ?
27. Rectangle $ABCD$ with $AB = 6$ and $BC = 7$ is inscribed in a semicircle such that CD lies on the diameter. If the area of the semicircle is $a\pi$, find a .
28. Suppose the median of the set $\{1, 2, 5, 5, 8, 9, 12, 15, x\}$ is one less than its mean. What is the sum of the possible values of x ?
29. Kathy is randomly sewing 6 buttons onto her shirt in a vertical line. She has three colors of buttons, red, orange, and yellow, and two of each. The probability that there will be at least one consecutive ordering of red, orange, yellow, on her shirt from top to bottom, can be expressed as $\frac{m}{n}$, where m, n are relatively prime positive integers. Find $m - n$.
30. Even-Steven's arch-nemesis, Odd-Todd, rigged a coin so that it would become heads 75% of the time. The two play a game together where they flip a coin 5 times. Odd-Todd is so overly confident he said that he would only win the game if he flipped 3 heads or 5 heads, as he loves odd numbers. The positive difference between the probabilities of either of them winning can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?
31. If Lance rolls 3 dice, the probability that all 3 numbers are prime but their sum is not prime can be expressed as $\frac{m}{n}$ for relatively prime positive integers m and n . What is $m + n$?
32. A healer and knight are battling a hydra with 20 heads. The healer can give the hydra exactly 4 heads each action. The knight can take exactly 7 heads off each action. The hydra will only be defeated if it has exactly 1 head left. If it has any less, it will immediately grow back to its original 20 heads state. What is the least amount of actions taken by the two to defeat the hydra?
33. If $32^x = 9^y$ and $z^x = 243^y$, what is $\sqrt[5]{z^2}$?
34. Terri loves playing tennis. Every day, she plays 4 matches in her division, and in any match, the probability of her winning is 80%. Tate, who is in a different division, plays 3 matches every day, and for any match, his probability of winning is 50%. The probability that Terri and Tate both win exactly 2 matches in their respective divisions can be expressed in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?
35. Ishan is trying to find out Ayush's birthday through only yes or no questions, which he will answer truthfully and correctly. He knows the month and the year, but does not the date. Ishan asks "Is the number prime?" to which Ayush responds yes. Ishan asks "Can the number be represented as the sum of two non-negative perfect squares?" to which Ayush responds yes. Ishan asks "Does the sum of the digits have an even number of positive divisors?" to which Ayush responds no. When Ishan tries to ask another

question, Ayush stops him and tells him to think, at which point he correctly states the number. What is the number?

36. A positive integer n is called a *threehugger* if exactly 3 positive integers divide n . How many threehuggers are between 333 and 2023, inclusive?
37. A regular tetrahedron and a cube have the same volume. If the ratio of the tetrahedron's surface area to the cube's surface area can be written as \sqrt{a} , what is a ?
38. How many integers, between 1 and 1295 (inclusive), have an even number of non-zero digits when expressed in base 6?
39. The integers from 1 to 29 are written in binary (base-2) notation, and those numbers are treated as though they were in base ten. Find their sum.
40. Pollo and Chicken are trapped in a rectangular pen of dimensions $4\sqrt{2}$ by 4, and are tied by a string of length 4 to opposite vertices of the pen. If the fraction of Pollo's roaming area in which their individually roaming areas overlap can be expressed as $\frac{a}{b} - \frac{\sqrt{c}}{\pi}$, for positive integers a, b, c , with a and b relatively prime, find $a + b + c$.