Answer Sheet

Name: ______________________ **ID**: ____________

1.	11.	21.	31.
2.	12.	22.	32.
3.	13.	23.	33.
4.	14.	24.	34.
5.	15.	25.	35.
6.	16.	26.	36.
7.	17.	27.	37.
8.	18.	28.	38.
9.	19.	29.	39.
10.	20.	30.	40.

FOR GRADER USE ONLY:

Score 1	Score 2	Score 3	Score 4

Total Score:

Joe Holbrook Memorial Math Competition

6th Grade

October 16, 2022

General Rules

- You will have **75 minutes** to solve **40 questions**. Your score is the number of correct answers.
- Only answers recorded on the answer sheet will be graded.
- This is an individual test. Anyone caught communicating with another student will be removed from the exam and their score will be disqualified.
- Scores will be posted on the website. Please do not forget your ID number, as that will be the sole means of identification for the scores.
- You may use the following aids:
  - Pencil or other writing utensil
  - Eraser
  - Blank scrap paper
- You may not use the following aids:
  - The Internet
  - Books or other written sources
  - Other people
  - Calculator or other computing device
  - Compass
  - Protractor
  - Ruler or straightedge

Other Notes

- All answers are integers. Make sure you do not make any mistakes when writing your answers, as you will not be given credit if you do so.
- You do not need to write units in your answers.
- Ties will be broken by the number of correct responses to questions 31 through 40. Further ties will be broken by the number of correct responses in the last five questions.

1. If 24% of x equals 30% of y, what percent of x is y?

2. Find the smallest odd positive prime p such that $p^2 + 4$ is not prime.

3. Timmy makes 10 paper cutouts every hour and Shawn makes 50% more paper cutouts every hour than Timmy. How many more hours than Shawn does Timmy have to work, if they both make 1200 cutouts?

4. Phoenix has a 13" x 4" sheet of chocolate cake which she wants to cut into 2" x 2" pieces. At most how many pieces can Phoenix get?

5. Lance has written 12 problems, but now he has to write the solutions to them! Each problem is rated an integer difficulty from 1 to 10, and a problem of difficulty i takes him i minutes to write a solution for. If his problems' average difficulty is 6, how many minutes will it take him to write all of his solutions?

6. Eshaan listens to 14 songs a day. How many whole days will it take for him to listen to 293 songs?

7. Rose has a 20% off coupon and a \$10 off coupon for a certain clothing store. She wants to apply both of these coupons (which the store allows) for a \$100 pair of heels. What is the minimum amount Rose can pay for these heels?

8. King Henry VIII challenges his court to guess his favorite natural number. A wise nobleman is able to correctly determine his favorite natural number after asking three questions, all of which the king answers correctly: is it 1-digit, is it prime, and is it odd? What is the king's favorite natural number?

9. Tasha loves collecting sea shells. One day she finds 5 seashells. Every day after that (starting on the second day) she makes it a goal to collect 4 more seashells than the previous day. After 10 days, how many seashells does Tasha have in total?

10. Let a be a positive integer such that there are 9 positive multiples of a less than 100. Let b be a positive integer such that there are 19 positive multiples of b less than 100. How many positive multiples of ab are less than 100?

11. Justin wants to test if his new computer is working. He selects an arbitrary real number k from 0 to 10 inclusive, and asks his computer to find the root to the equation $3x + k = 0$. The probability that the output of the computer is less than $-\frac{2}{5}$ can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?

12. I have 4 cupcakes. Every day starting from day 2, I double the amount of cupcakes I have and then eat three. On the first day I start with 4 cupcakes, then on the second I have 5, on the third I have 7, and so on. How many cupcakes will I have on the tenth day?

13. The road between City A and City B is 450 miles long, and cars travel at 60 mph (miles per hour) on the road. Alternatively, there is a flight from City A to City B that travels at 240 mph along a more direct 400 mile route, but it takes two hours to get to the airport and board the flight. By how many minutes is flying faster than driving?

14. Alice and Bob take 3 hours to mow the lawn together, but Alice can do it alone in 5 hours. Assuming that each person always works at a constant speed, how many minutes does it take Bob to mow the lawn?

15. Tasha is having a tea party. She buys a large box of biscuits and wants to distribute the biscuits equally among her guests. At first, she only invited five people to the party and, after distributing the biscuits, had 2 left over. Tasha then invites three more people, after which distributing the biscuits would result in there being 5 biscuits left over. Finally, one person tells Tasha that they cannot make it to the party, so the remaining people get an equal amount of biscuits and there are 4 biscuits left over. What is the least possible number of biscuits in the box that Tasha bought?

16. If a rhombus has a perimeter of 40 and an area of 96, what is the sum of the lengths of the diagonals?

17. Mayer builds a fence next to a wall longer than 300 feet, such that the enclosed area is a rectangle. If she has 300 feet of fencing, what is the maximum area in square feet she can fence in?

18. The sum of all but one angle of a convex polygon (a polygon with no angle over 180 degrees) is 2022 degrees. What is the measure of the missing angle in degrees?

19. I have a square piece of paper $ABCD$, with side length 30. I cut it into an isosceles right triangle EAF and a pentagon $EBCDF$, with E on $\overline{AB}$ and F on $\overline{AD}$. If the area of the triangle is one half of the area of the pentagon, what is AE^2?

20. A woman is standing on a field of corn is wearing a circular sombrero of radius 4 feet. Unfortunately, the hat blocks the sunlight so effectively that the corn directly under it dies instantly. If the woman walks along the circumference of a circle of radius 6 feet, the area of dead corn can be written as $a\pi$. What is a?

21. $\triangle ABC$ has side lengths $AB = 6, BC = 8$, and $AC = 10$. Let the midpoint of $\overline{AC}$ be M. If D is on $\overline{BC}$ and $\overline{MD} \perp \overline{AC}$, then the area of $\triangle MDC$ can be expressed as $\frac{m}{n}$ for relatively prime positive integers m and n. What is $m+n$?

22. In how many ways can 2022 be written as the sum of consecutive integers?

23. Andrea and Anthony are running together to be on a committee chosen randomly from 100 people, with only a president and vice president. Label any scenario in which Andrea and Anthony are vice-president and president, irrespective of which one of them gets which position *ideal*. The probability that the committee ends up being *ideal* can be written $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m+n$?

24. Find the sum of all positive integers N such that the second largest factor of N is 15 times its second smallest factor.

25. The city of Lattice has 26 evenly spaced streets, labeled A-Z in order, going north to south, and 26 evenly spaced avenues, labeled 1-26 in order, going east to west. Intersections are labeled with the street and avenue in order (e.g. Z1 is the intersection of Z Street and 1 Avenue). Cycling is only allowed along the streets and avenues. If the cycling distance from C9 to F13 is exactly 1 mile and the cycling distance between J11 to P12 is 1,420 yards, what is the cycling distance from A1 to B2 in feet? Note a mile is 1760 yards.

26. Compute the number of ordered pairs of positive integers (a, b) of integers satisfying $\text{lcm}(a, b) = 60$.

27. A point (x, y) is randomly picked inside a trapezoid with vertices $(2, 0), (4, 0), (6, 2)$ and $(0, 2)$. The probability that $y < \frac{x}{3}$ can be expressed as $\frac{m}{n}$ where m and n are relatively prime positive integers. What is $m+n$?

28. Unlike our base-10 numeral system, the Mayan numeral system uses base-20: it has 20 digits representing remainders of 0 through 19. Find the least number of digits needed to write 10^{20} in Mayan numerals.

29. Let $p(n)$ denote the product of the digits of n. For how many two digit numbers n does repeatedly applying the function p (in the form of $p(p(\cdots p(n)\cdots)))$ eventually result in zero?

30. Amir is at $(0, 0)$ in the coordinate plane and is walking towards his house, which is at $(8, 8)$, such that all of his steps are either up or right and of length 1. Unfortunately, construction is being done at every point (a, b) where both a and b are odd, so Amir cannot travel to any of these points along his path. How many different routes can Amir take?

31. Tasha is trying to find the ideal size for a pet rock that she wants roll through a gap created by a larger boulder, the ground, and a mountain. She draws the boulder as a circle with radius 5 feet, the ground as a horizontal line tangent to the circle, and the mountain as a vertical line also tangent to the circle. If Tasha's pet rock is also represented as a circle and the maximum possible radius of the pet rock is $a - \sqrt{b}$ where a, b are integers, what is $2a$?

32. Elsa and Anna are playing a game where they roll three fair six-sided dice. Elsa gets a point if the first two rolls add up to the third roll while Anna gets a point if the first two rolls multiply to the number on the third roll. If they play the game 144 times, the expected difference in their final point totals can be expressed as $\frac{m}{n}$ where m and n are relatively prime positive integers. What is $m+n$?

33. Kai is trying to type his name, but even though he has resorted to typing with only the letters A through L, he still makes "erorrs". In any letter position, the probability of typing any letter that actually appears in his name is twice that of typing a given letter that is not in his name; for example, K's probability is twice that of D. Given that he manages to type at least one letter in its correct position, the probability that he spells his own name correctly is $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m+n$?

34. BCA is hosting a tournament in which 32 players compete in knockout-round format. The players are all given a ranking between 1 and 32, where 1 is the best and 32 is the worst ranking. However, BCA rigs it so that the tournament plays out in the following way:

- Players 1 and 2 must be in the final round
- Players $1-4$ must be in the semi-finals
- Players $1-8$ must be in the quarter-finals
- Players $1-16$ must be in the octo-finals

If the better-ranked player always wins in a matchup, let N be the number of ways a tournament bracket can be filled out. What is the largest integer k such that 2^k divides N? A tournament is identical if and only if all the matchups are the same.

35. A number n has the property that n^2 is divisible by 2023 but n is not. Find the sum of all possible remainders of n when divided by 2023.

36. Two circles of radius 6 have centers O_1 and O_2 such that $O_1O_2 = 6\sqrt{3}$. If the area of the region contained in both circles can be expressed as $a\pi + b\sqrt{c}$ for integers a, b, and c, with c nonnegative and squarefree, find $a + b + c$.

37. Let $\lfloor x \rfloor$ be the greatest integer less than or equal to x (e.g. $\lfloor \pi \rfloor = 3, \lfloor 9.98 \rfloor = 9$). The fraction

$$\frac{\lfloor \sqrt[5]{1} \rfloor \cdot \lfloor \sqrt[5]{3} \rfloor \cdot \lfloor \sqrt[5]{5} \rfloor \ldots \lfloor \sqrt[5]{2021} \rfloor}{\lfloor \sqrt[5]{2} \rfloor \cdot \lfloor \sqrt[5]{4} \rfloor \cdot \lfloor \sqrt[5]{6} \rfloor \ldots \lfloor \sqrt[5]{2022} \rfloor}.$$

can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?

38. Let r and s be the solutions to the quadratic equation $x^2 - 35x + 49 = 0$. Given that r and s are positive real numbers, the value of $\sqrt{r} + \frac{1}{r} + \sqrt{s} + \frac{1}{s}$ can be expressed in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?

39. Due to Boss Math Camp's "no technology, no fun, no life" policy, Maia has a $\frac{1}{3}$ chance of going crazy each day. However, every day that she doesn't go crazy, her math knowledge increases by 5 potato points. The expected increase in her math knowledge in potato points, before she goes crazy, is E. Find E.

40. Nikhil and Jaiden are avid gamblers, and they decide to place bets against each other to perhaps earn some money. On the number line from 1 to 18, Jaiden places an ant on 13. Every second, the ant changes its position by $+1$ with probability $\frac{1}{2}$ and -1 otherwise. If the ant reaches position 1, Jaiden must pay Nikhil 17 dollars, and if the ant reaches position 18, Nikhil must pay Jaiden 17 dollars. The bet is over once the ant reaches either position 1 or 18. What is Jaiden's expected monetary gain, in dollars?