Answer Sheet

Name: ______________________ **ID**: ____________

1.	11.	21.	31.
2.	12.	22.	32.
3.	13.	23.	33.
4.	14.	24.	34.
5.	15.	25.	35.
6.	16.	26.	36.
7.	17.	27.	37.
8.	18.	28.	38.
9.	19.	29.	39.
10.	20.	30.	40.

FOR GRADER USE ONLY:

Score 1	Score 2	Score 3	Score 4

Total Score:

Joe Holbrook Memorial Math Competition

8th Grade

October 16, 2022

General Rules

- You will have **75 minutes** to solve **40 questions**. Your score is the number of correct answers.
- Only answers recorded on the answer sheet will be graded.
- This is an individual test. Anyone caught communicating with another student will be removed from the exam and their score will be disqualified.
- Scores will be posted on the website. Please do not forget your ID number, as that will be the sole means of identification for the scores.
- You may use the following aids:
  - Pencil or other writing utensil
  - Eraser
  - Blank scrap paper
- You may not use the following aids:
  - The Internet
  - Books or other written sources
  - Other people
  - Calculator or other computing device
  - Compass
  - Protractor
  - Ruler or straightedge

Other Notes

- All answers are integers. Make sure you do not make any mistakes when writing your answers, as you will not be given credit if you do so.
- You do not need to write units in your answers.
- Ties will be broken by the number of correct responses to questions 31 through 40. Further ties will be broken by the number of correct responses in the last five questions.

1. If 24% of x equals 30% of y, what percent of x is y?

2. Find the smallest odd positive prime p such that $p^2 + 4$ is not prime.

3. Tyrone, Uniqua, and Pablo each made their own snowball making machine to have an epic snowball fight. If Tyrone and Uniqua's machines work together, they can make 32 snowballs in one hour. If Uniqua and Pablo's machines work together, they can make 45 snowballs in one hour. If all three machines work together, they can make 57 snowballs in one hour. How many snowballs can Tyrone and Pablo's machines make in one hour if they work together?

4. The road between City A and City B is 450 miles long, and cars travel at 60 mph (miles per hour) on the road. Alternatively, there is a flight from City A to City B that travels at 240 mph along a more direct 400 mile route, but it takes two hours to get to the airport and board the flight. By how many minutes is flying faster than driving?

5. The City of Hackensack has decided to open a zoo with only snakes, chickens, and parrots. When Ann visited the zoo, she counted 24 heads and 38 legs. If chickens and parrots both have two legs, while snakes have none, how many snakes are there at the Hackensack Zoo?

6. It is a very windy day and Uniqua is flying in her airplane. She flies north against the wind for 1 hour and then flies south with the wind for 2 hours. Then, she realizes that she is 1200 miles south of where she started. What is the wind speed (in mph) if it takes Uniqua 3 hours to fly back to where she started?

7. Tyron has an apple tree with 40 apples. The Apple Man comes along and tells Tyron three true statements:

   - The number of rotten apples in the tree is a perfect square.
   - The number of non-rotten apples shares a prime factor with 10.
   - There are an odd number of non-rotten apples.

   How many apples in the apple tree are not rotten?

8. A positive integer is called *fishy* if all of its digits are composite, but it is prime. Find the sum of all fishy numbers less than 100.

9. Point A is located at $(2, 4)$ and point B is located at $(8, 12)$. Jack walks from A to B in a straight line. Jane walks from A to B in a semicircular path. The absolute difference between the distances they traveled can be expressed as $a\pi - b$, where a and b are positive integers. Compute $a + b$.

10. Three circles of radius 2 are mutually externally tangent. If the area between them can be represented as $\sqrt{a} - b\pi$, what is $a + b$?

11. I have a square piece of paper $ABCD$, with side length 30. I cut it into an isosceles right triangle EAF and a pentagon $EBCDF$, with E on $\overline{AB}$ and F on $\overline{AD}$. If the area of the triangle is one half of the area of the pentagon, what is AE^2?

12. Let $s(n)$ be the sum of digits of n. What is the smallest positive integer n such that $s(s(n)) = 11$?

13. Mr. Nelson writes down two positive integers, A and B, on a whiteboard and asks Matt what the sum of the two numbers are. Matt says that the sum is $94,395$, but it turns out that Matt misread every 1 as a 7 and every 7 as a 1 (his arithmetic is otherwise perfect). What is the greatest possible correct sum of A and B?

14. If a rhombus has a perimeter of 40 and an area of 96, what is the sum of the lengths of the diagonals?

15. Find the sum of all positive integers N such that the second largest factor of N is 15 times its second smallest factor.

16. Suppose that x is a 3 digit number (with the first digit non zero). We say x has a "nice average" if it is the average of the six numbers formed by the permutations of its digits. An example of a number which has a nice average is 629, because 629 is the average of the numbers $629, 692, 269, 296, 926$, and 962. The sum of all 3 digit numbers which have a "nice average" is S. Compute the remainder when S is divided by 37.

17. Rectangle $ABCD$ with $AB = 6$ and $BC = 7$ is inscribed in a semicircle such that CD lies on the diameter. If the area of the semicircle is $a\pi$, find a.

18. A point (x, y) is randomly picked inside a trapezoid with vertices $(2, 0), (4, 0), (6, 2)$ and $(0, 2)$. The probability that $y < \frac{x}{3}$ can be expressed as $\frac{m}{n}$ where m and n are relatively prime positive integers. What is $m + n$?

19. Unlike our base-10 numeral system, the Mayan numeral system uses base-20: it has 20 digits representing remainders of 0 through 19. Find the least number of digits needed to write 10^{20} in Mayan numerals.

20. There are 6 soccer teams in the new BCA league. Each team will play every other team twice. A team gets 3 points for a win, 1 point for a tie, and 0 points for a loss. Let M, N be the maximum and minimum sum of all the points for all the teams after every game is played, respectively. What is $M - N$?

21. Let $\lfloor x \rfloor$ be the greatest integer less than or equal to x (e.g. $\lfloor \pi \rfloor = 3, \lfloor 9.98 \rfloor = 9$). The fraction

$$\frac{\lfloor \sqrt[5]{1} \rfloor \cdot \lfloor \sqrt[5]{3} \rfloor \cdot \lfloor \sqrt[5]{5} \rfloor \ldots \lfloor \sqrt[5]{2021} \rfloor}{\lfloor \sqrt[5]{2} \rfloor \cdot \lfloor \sqrt[5]{4} \rfloor \cdot \lfloor \sqrt[5]{6} \rfloor \ldots \lfloor \sqrt[5]{2022} \rfloor}.$$

can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?

22. Amir is at $(0, 0)$ in the coordinate plane and is walking towards his house, which is at $(8, 8)$, such that all of his steps are either up or right and of length 1. Unfortunately, construction is being done at every point (a, b) where both a and b are odd, so Amir cannot travel to any of these points along his path. How many different routes can Amir take?

23. An equilateral triangle is inscribed in a circle of radius 6. If a dime of radius 1 is thrown completely inside the circle, the probability that the dime also lands completely inside the triangle can be expressed as $\frac{a\sqrt{b}}{c\pi}$, where b is square-free and a, c are relatively prime positive integers. Find $a + b + c$.

24. Tasha is trying to find the ideal size for a pet rock that she wants roll through a gap created by a larger boulder, the ground, and a mountain. She draws the boulder as a circle with radius 5 feet, the ground as a horizontal line tangent to the circle, and the mountain as a vertical line also tangent to the circle. If Tasha's pet rock is also represented as a circle and the maximum possible radius of the pet rock is $a - \sqrt{b}$ where a, b are integers, what is $2a$?

25. 6 doughnuts—2 jelly-filled, 2 chocolate, and 2 glazed—are to be divided up among Adisa, Batu, and Catalina so that everyone receives exactly 2 doughnuts. If two doughnuts of the same flavor are identical, in how many ways can the treats be distributed if each person refuses to get both of the glazed doughnuts?

26. BCA is hosting a tournament in which 32 players compete in knockout-round format. The players are all given a ranking between 1 and 32, where 1 is the best and 32 is the worst ranking. However, BCA rigs it so that the tournament plays out in the following way:

    - Players 1 and 2 must be in the final round
    - Players $1 - 4$ must be in the semi-finals
    - Players $1 - 8$ must be in the quarter-finals
    - Players $1 - 16$ must be in the octo-finals

    If the better-ranked player always wins in a matchup, let N be the number of ways a tournament bracket can be filled out. What is the largest integer k such that 2^k divides N? A tournament is identical if and only if all the matchups are the same.

27. How many ways can Nikhil place 8 disks of different radii on 3 distinct poles so no pole is empty and no disk is above a disk of smaller radius than itself?

28. Pollo and Chicken are trapped in a rectangular pen of dimensions $4\sqrt{2}$ by 4, and are tied by a string of length 4 to opposite vertices of the pen. If the fraction of Pollo's roaming area in which their individually roaming areas overlap can be expressed as $\frac{a}{b} - \frac{\sqrt{c}}{\pi}$, for positive integers a, b, c, with a and b relatively prime positive integers, find $a + b + c$.

29. Clumsy Josh is thinking of a 4-digit number. He claims that when he adds up the four three-digit numbers obtained by crossing out each digit of the original number and reading the other three digits in order, he gets 3308. Julia, however, quickly realizes that Josh made a mistake when he added up the four three-digit numbers. Knowing that Josh's actual sum is less than 3308, what is the largest four-digit number Josh could have been thinking of?

30. A number n has the property that n^2 is divisible by 2023 but n is not. Find the sum of all possible remainders of n when divided by 2023.

31. How many ways are there for Jim to distribute his 13 indistinguishable apples amongst his 4 friends so that each of Jim's friends gets at least one apple and at least two people get the same number of apples?

32. Find the integer closest to the area enclosed by the equation $|x^2 + 2x + 6| + |y^2 - 7y + 14| = 13$.

33. How many integers, between 1 and 1295 (inclusive), have an even number of non-zero digits when expressed in base 6?

34. Let r and s be the solutions to the quadratic equation $x^2 - 35x + 49 = 0$. Given that r and s are positive real numbers, the value of $\sqrt{r} + \frac{1}{r} + \sqrt{s} + \frac{1}{s}$ can be expressed in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

35. Let $ABCDE$ be a convex pentagon with all five sides having length 10, and $AB \parallel DE$. The range of possible areas of $ABCDE$ is $(m, n]$. If $m + n$ can be expressed as $p + q\sqrt{r}$ for positive integers p, q, r with r squarefree, find $p + q + r$.

36. How many subsets A of $S = \{1, 2, 3, \ldots, 10\}$ satisfy $\gcd(A) = \gcd(S - A)$, where $S - A$ is the set of elements in S that are not in A?. (The gcd of the empty set would be 1, in this case).

37. Jaiden has recently come into possession of an isosceles right triangle. He randomly picks a point inside the triangle and folds all three vertices of the triangle onto the point. If p is the probability that the 3 folds do not overlap, find the closest integer to $100p$.

38. For all positive integers $a, b, c < 1500$ where

$$1000 < \text{lcm}(7a, \gcd(17b, \text{lcm}(2023, c))) < 1500,$$

what is the maximum value of $a + b + c$?

39. Eshaan is at $(0, 0)$ and is going to BCA, which is at $(10, 10)$. How many paths can he take such that his first and last move are in different directions and he takes an even number of right turns? Eshaan can only travel one unit up or one unit right.

40. There are ten people standing equidistant along the perimeter of a circle. Each person randomly chooses someone (possibly themselves!) to "follow". Every second, each person moves to the position that the person they were following was previously at; that is, if person a is at position x at second t, anyone following a would be at position x at second $t + 1$. The probability that everyone ends up in their original spot after 120 seconds is equal to $\frac{m}{n}$ where m and n are relatively prime positive integers. What is the sum of the number of positive factors of m and the number of positive factors of n?