Joe Holbrook Memorial Invitational Competition

4th Grade

March 19, 2023

General Rules

- You will have **90 minutes** to solve **16 questions**. Your score is the number of correct answers.

- Only answers recorded on the answer sheet will be graded.

- This is an individual test. Anyone caught communicating with another student will be removed from the exam and their score will be disqualified.

- You may use the following aids:

    - Pencil or other writing utensil
    - Eraser
    - Blank scrap paper

- You may not use the following aids:

    - Other people
    - Calculator or other computing device
    - Compass
    - Protractor
    - Ruler or straightedge

Other Notes

- Write legibly. If the graders cannot read your answer, you will be given no credit for that question.

- **All answers are integers.**

- You do not need to write units in your answers.

- Ties will be broken by the number of correct responses to the last 4 problems. Further ties will be broken by the number of correct responses to the previous 4 problems, etc.

- Keep in mind that the JHMIC is a difficult contest and very different from school assessments. If you even get a few questions right, you should feel proud of yourself!

1. Compute $\left(2 + \dfrac{9}{2+1}\right) \times \left(8 + \dfrac{63}{8+1}\right)$.

2. There is an equilateral triangle with side length 12 and a square with side length 9. What is the ratio of the equilateral triangle's perimeter to the square's perimeter?

3. In my drawer, I have 17 unique pairs of socks. If I randomly take socks out of my drawer, how many must I take to guarantee I have a matching pair?

4. Scoorge McDuck accidentally dropped a money bag! Ma Beagle starts 100 meters east of the bag and Flintheart Glomgold starts 150 meters west of the bag. If Glomgold runs at 20 meters per second and reaches the bag 5 seconds later than Ma Beagle, how fast was Ma Beagle going in meters per second?

5. Charles is a very indecisive person. He has to choose one out of three books to read. There are 11 copies of the first book, and 15 copies of another. If there are a total of 50 books and Charles is equally likely to choose any of the 50 books to read, the probability that Charles chooses the third type of book is $\dfrac{a}{b}$ where a and b are relatively prime positive integers. What is $a + b$?

6. Violet has been rolling on a slope at 50 miles per hour for 5 minutes. If the slope is 10 miles long, what does her average speed need to be during the rest of her trip so that she reaches the end of the slope in exactly 10 minutes?

7. The number $1\,2\,3\,9\,1\,0\,2\,4\,\underline{X}$, where X represents a single digit, is divisible by both 2 and 3. What is the sum of all possible values of X?

8. Call a natural number n "magic" if it is a perfect square, divisible by 6, and a multiple of 5. How many magic numbers are there that are less than 10000?

9. Square $ABCD$ has side length 4 and a point P inside it. What is the area of triangle ABP plus the area of CDP?

10. Peter has a collection of foxes and rabbits. He says three statements:

    - The number of foxes is 20 more than the number of rabbits.
    - The number of foxes is equal to the number of rabbits squared.
    - The number of foxes is equal to six times the number of rabbits.

    However, it is revealed that one of Peter's statements is false, while the other two are true. What is the maximum number of foxes Peter has?

11. A number is called "alternative" if it is positive and its digits alternate between two distinct values such that the preceding and succeeding digit of a number are necessarily the same. As an example, 3737 is an alternative number, but 2494 is not. How many six digit alternative numbers are divisible by 4 if numbers can begin with 0?

12. Anna, Bob, and Carol play a game where they each select a unique number from $\{1, 2, 3, \ldots, 10\}$. They select $a, b,$ and c, respectively. They then ask questions about each other's numbers, to which their responses are truthful.

    - Anna asks Bob, "Is your number prime?" to which Bob replies No.
    - Bob asks Carol, "Is your number greater than 5?" to which Carol replies Yes.
    - Carol asks Anna, "Is your number a factor of 12?" to which Anna replies Yes.

    How many possible triples (a, b, c) are there?

13. For how many positive integers n less than 1000 is $\operatorname{lcm}(90, n) = 5n$?

14. Helen wanted to go to Emmy's birthday party 7 blocks east and 6 blocks north. She also wants to buy some instruments as a gift, but can only find one xylophone dealer 2 blocks east and 3 blocks north of her starting point. She then decides to buy the rest of the instruments at an instrument shop 5 blocks east and 4 blocks north of her starting point, and finally goes to the party. How many ways can Helen get to Emmy's birthday party passing through the two points if she can only go 1 block north or east at a time?

15. The integer n is the smallest positive multiple of 24 such that each digit is either 2 or 3. Compute $\dfrac{n}{24}$.

16. How many 5 letter sequences of the letters $A, B, C,$ and D have an even number of A's? Note that zero is an even number.