Joe Holbrook Memorial Invitational Competition

5th Grade

March 19, 2023

General Rules

- You will have **90 minutes** to solve **16 questions**. Your score is the number of correct answers.
- Only answers recorded on the answer sheet will be graded.
- This is an individual test. Anyone caught communicating with another student will be removed from the exam and their score will be disqualified.
- You may use the following aids:
  - Pencil or other writing utensil
  - Eraser
  - Blank scrap paper
- You may not use the following aids:
  - Other people
  - Calculator or other computing device
  - Compass
  - Protractor
  - Ruler or straightedge

Other Notes

- Write legibly. If the graders cannot read your answer, you will be given no credit for that question.
- **All answers are integers.**
- You do not need to write units in your answers.
- Ties will be broken by the number of correct responses to the last 4 problems. Further ties will be broken by the number of correct responses to the previous 4 problems, etc.
- Keep in mind that the JHMIC is a difficult contest and very different from school assessments. If you even get a few questions right, you should feel proud of yourself!

1. Define $a \oplus b$ to be $a + \dfrac{b}{a+1}$. Compute $(2 \oplus 9) \times (8 \oplus 63)$.

2. There is an equilateral triangle with side length 12 and a square with side length 9. What is the ratio of the equilateral triangle's perimeter to the square's perimeter?

3. In my drawer, I have 17 unique pairs of socks. If I randomly take socks out of my drawer, how many must I take to guarantee I have a matching pair?

4. Charles is a very indecisive person. He has to choose one out of three books to read. There are 11 copies of the first book, and 15 copies of another. If there are a total of 50 books and Charles is equally likely to choose any of the 50 books to read, the probability that Charles chooses the third type of book is $\dfrac{a}{b}$ where a and b are relatively prime. What is $a + b$?

5. The number $\underline{1}\,\underline{2}\,\underline{3}\,\underline{9}\,\underline{1}\,\underline{0}\,\underline{2}\,\underline{4}\,\underline{X}$, where X represents a single digit, is divisible by both 2 and 3. What is the sum of all possible values of X?

6. Square $ABCD$ has side length 4 and a point P inside it. What is the area of triangle ABP plus the area of CDP?

7. Violet has been rolling on a slope at 50 miles per hour for 5 minutes. If the slope is 10 miles long, what does her average speed need to be during the rest of her trip so that she reaches the end of the slope in exactly 10 minutes?

8. Call a natural number n "magic" if it is a perfect square, divisible by 6, and a multiple of 5. How many magic numbers are there that are less than 10000?

9. Peter has a collection of foxes and rabbits. He says three statements:

   - The number of foxes is 20 more than the number of rabbits.
   - The number of foxes is equal to the number of rabbits squared.
   - The number of foxes is equal to six times the number of rabbits.

   However, it is revealed that one of Peter's statements is false, while the other two are true. What is the maximum number of foxes Peter has?

10. A number is called "alternative" if its digits alternate between two distinct values such that the preceding and succeeding digit of a number are necessarily the same. As an example, 3737 is an alternative number, but 2494 is not. How many six digit alternative numbers are divisible by 4 if numbers can begin with 0?

11. For how many positive integers n less than 1000 is $\text{lcm}(90, n) = 5n$?

12. Helen wanted to go to Emmy's birthday party 7 blocks east and 6 blocks north. She also wants to buy some instruments as a gift, but can only find one xylophone dealer 2 blocks east and 3 blocks north. She then decides to buy the rest of the instruments at an instrument shop 5 blocks east and 4 blocks north of her starting point, and finally goes to the party. How many ways can Helen get to Emmy's birthday party passing through the two points if she can only go 1 block north or east at a time?

13. Four circles of radius 4 are internally tangent to a circle of radius r, with each of the smaller circles externally tangent to two others. If r is written as $a + b\sqrt{c}$ for positive integers a, b, c with c squarefree, then find $a + b + c$.

14. The integer n is the smallest positive multiple of 24 such that each digit is either 2 or 3. Compute $\dfrac{n}{24}$.

15. Let $y > 1$. Given $y + \dfrac{1}{y} = 34$, $y - \dfrac{1}{y}$ can be expressed as $a\sqrt{b}$, where b is not divisible by the prime factor of any square. Find $a + b$.

16. Consider the set of all parallelograms that have the points $(1, 2), (2, 5), (-5, 2)$ among its vertices. Compute the sum of the areas of all of these parallelograms.