Joe Holbrook Memorial Invitational Competition

8th Grade

March 19, 2023

General Rules

- You will have **90 minutes** to solve **16 questions**. Your score is the number of correct answers.
- Only answers recorded on the answer sheet will be graded.
- This is an individual test. Anyone caught communicating with another student will be removed from the exam and their score will be disqualified.
- You may use the following aids:
  - Pencil or other writing utensil
  - Eraser
  - Blank scrap paper
- You may not use the following aids:
  - Other people
  - Calculator or other computing device
  - Compass
  - Protractor
  - Ruler or straightedge

Other Notes

- Write legibly. If the graders cannot read your answer, you will be given no credit for that question.
- **All answers are integers.**
- You do not need to write units in your answers.
- Ties will be broken by the number of correct responses to the last 4 problems. Further ties will be broken by the number of correct responses to the previous 4 problems, etc.
- Keep in mind that the JHMIC is a difficult contest and very different from school assessments. If you even get a few questions right, you should feel proud of yourself!

1. In my drawer, I have 17 unique triplets of socks. If I randomly take socks out of my drawer, how many must I take to guarantee I have a matching triplet?

2. The number $\underline{1}\,\underline{2}\,\underline{3}\,\underline{9}\,\underline{1}\,\underline{0}\,\underline{2}\,\underline{4}\,\underline{X}$, where X represents a single digit, is divisible by both 2 and 3. What is the sum of all possible values of X?

3. Square $ABCD$ has side length 4 and a point P inside it. What is the area of triangle ABP plus the area of CDP?

4. A number is called "alternative" if its digits alternate between two distinct values such that the preceding and succeeding digit of a number are necessarily the same. As an example, 3737 is an alternative number, but 2494 and 2222 is not. How many six digit alternative numbers are divisible by 4 if numbers can begin with 0?

5. How many 5 letter sequences of the letters A, B, C, and D have an even number of A's? Note that zero is an even number.

6. Call a natural number n "magic" if it is a perfect square, divisible by 6, and a multiple of 5. How many magic numbers are there that are less than 10000?

7. Four circles of radius 4 are internally tangent to a circle of radius r, with each of the smaller circles externally tangent to two others. If r is written as $a + b\sqrt{c}$ for positive integers a, b, c with c squarefree, then find $a + b + c$.

8. Anna, Bob, and Carol play a game where they each select a unique number from $\{1, 2, 3, \ldots, 10\}$. They select a, b, and c, respectively. They then ask questions about each other's numbers, to which their responses are truthful.

   - Anna asks Bob, "Is your number prime?" to which Bob replies No.
   - Bob asks Carol, "Is your number greater than 5?" to which Carol replies Yes.
   - Carol asks Anna, "Is your number a factor of 12?" to which Anna replies Yes.

   How many possible triples (a, b, c) are there?

9. A positive integer is k-smooth if all of its prime divisors are less than or equal to k. What is the least common multiple of the 5-smooth numbers less than 100?

10. Consider the set of all parallelograms that have the points $(1, 2), (2, 5), (-5, 2)$ among its vertices. Compute the sum of the areas of all of these parallelograms.

11. Nikhil can paint a room in x hours, Jaiden can paint the same room in y hours, and working together, they can paint the same room in xy hours (where x and y are real numbers). What is the maximum amount of minutes it could take them to paint the room together?

12. The integer n is the smallest positive multiple of 24 such that each digit is either 2 or 3. Compute $\frac{n}{24}$.

13. How many pairs of positive integers (a, r) exist such that a and r divides 210^{17} and $\operatorname{lcm}(a, r^2) = \frac{a^2}{r}$?

14. How many distinct terms are there in the sequence $\left\lfloor \frac{n^2}{2022} \right\rfloor$ as n goes from 1 to 2022? ($\lfloor x \rfloor$ denotes the largest integer less than or equal to x. For example, $\lfloor 20.345 \rfloor = 20$).

15. There are 7 people in a circle passing one ball around. A person cannot pass the ball to someone adjacent to them, and a person cannot pass the ball to themself. How many ways can the ball be passed around so that the ball starts and ends at the same person and is passed a total of 7 times?

16. Let $ABCD$ be a trapezoid with $AB \parallel CD$ and $\angle ADC, \angle BCD < 90^\circ$. If the angle bisector $\angle ADC$ intersects side BC at its midpoint, $AD = 5$, and $BC = 7$, then the interval of possible areas for this trapezoid can be written as $(x, y]$. Given that x can be expressed as $\frac{a\sqrt{b}}{c}$, where b is squarefree and a and c are relatively prime, compute $a + b + c$.