Joe Holbrook Memorial Math Competition

4th Grade

October 22, 2023

General Rules

- You will have **75 minutes** to solve **40 questions**. Your score is the number of correct answers.
- Only answers recorded on the answer sheet will be graded.
- This is an individual test. Anyone caught communicating with another student will be removed from the exam and their score will be disqualified.
- Scores will be posted on the website. Please do not forget your ID number, as that will be the sole means of identification for the scores.
- You may use the following aids:
  - Pencil or other writing utensil
  - Eraser
  - Blank scrap paper
- You may not use the following aids:
  - The Internet
  - Books or other written sources
  - Other people
  - Calculator or other computing device
  - Compass
  - Protractor
  - Ruler or straightedge

Other Notes

- All answers are integers. Make sure you do not make any mistakes when writing your answers, as you will not be given credit if you do so.
- You do not need to write units in your answers.
- Ties will be broken by the number of correct responses to questions 31 through 40. Further ties will be broken by the number of correct responses in the last five questions.

1. Mittens has 50 Styrofoam packing peanuts. If she throws 37 packing peanuts at Bolt, how many does she have left?
2. Evaluate $\frac{111}{1+1+1}$.
3. A book has 20 chapters, each 7 pages long. If I am reading page 73, which chapter am I on?
4. How many inches longer is 7.7 feet than 2.2 feet?
5. Every day, the height of Mr.Tree increases by the same amount. Yesterday, Mr.Tree was 87 feet tall, and today, Mr.Tree is 100 feet tall. How tall, in feet, will Mr.Tree be tomorrow?
6. If I were to split 47 cookies evenly between me and my 5 friends so we each get a whole number of cookies, and keep the remaining cookies to myself, how many cookies would I have in the end?
7. The Evil Queen has 200 apples. If she poisons 30% of them, how many poisoned apples does she have?
8. What is the fewest number of coins required to make 37 cents from pennies, nickels, dimes, and quarters?
9. Calculate the product of the one-digit prime numbers.
10. Bob has 10 matching pairs of mittens in a basket, each of a different color. He has just woken up and randomly picks mittens out of the basket. How many does he have to pick to guarantee that he has a matching pair of mittens?
11. What is the value of $(2^2+0^2+2^2+3^2)^2 \cdot (2+0+2+3)$ where for any number n, $n^2 = n \times n$?
12. Sceptile has started a lemonade stand! She currently sets a price of 60 cents per cup. If she raised the price to 70 cents per cup, she would sell 3 fewer cups per day, but the amount of money she makes each day would not change. How much money, in cents, does Sceptile's make each day?
13. Alice is trying to guess Bob's secret number. She knows it is an odd prime that is one more than triple a square number. What is the smallest possible value for Bob's secret number if it is not 13? A square number, n, is a number such that $n = p \times p$ for some number p.
14. The Cookie Monster is willing to trade 1 cookie for 3 carrots, or 3 cookies for 1 carrot. If Elmo has 30 cookies and 10 carrots, he can get a maximum of A carrots or a maximum of B cookies by trading with the Cookie Monster. Assuming the Cookie Monster has an infinite supply of cookies and carrots to trade, find $A - B$.
15. A square of side length 420 is divided into 49 smaller squares each with the same side length. What is the side length of one of the 49 squares?
16. Alyssa is 31 years old, and her son James is 7 years old. In how many years will Alyssa be twice as old as James?
17. Carl is deciding what he wants to wear to school. He has 3 shirts, 4 pairs of pants, and 2 hats to choose from. If he has to wear 1 shirt and 1 pair of pants but can choose whether or not to wear a hat, how many combinations of clothes can he wear?
18. Let the answer to this question be x. Given that x is nonzero, What is the value of $\frac{x \times x}{101}$?
19. Out of a group of birds, exactly $\frac{1}{3}$ can fly and the rest cannot. If 5 of the birds that cannot fly are penguins, what is the minimum possible number of birds in the group?
20. The Caligari Carnival sells tickets at a price of 7 dollars for adults and 4 dollars for children. If they make 453 dollars from a show with an audience of 78 people, how many children bought tickets?
21. Devin's favorite number has a remainder of 3 when divided by 4 and a remainder of 4 when divided by 5. If Devin's favorite number is positive, what is the smallest possible value of his favorite number?
22. Seedot folds a 6×6 piece of origami paper along its diagonal. He then rips a 2×2 square out of the right-angled corner of the resulting triangle. What is the area of the (now ripped) origami paper when unfolded?
23. A number is called *powerful* if it only consists of digits $1, 2, 4$, or 8. How many four-digit numbers are powerful?

24. Jack and Caleb both draw squares. No matter how large Jack draws his square, Caleb's square will always have 9 times the area. If Jack's square has a side length of 3 cm, how long will one side of Caleb's square be in centimeters?

25. How many positive whole numbers that are less than 2023 have an even number of divisors?

26. In a square with side length 1, a line is drawn through the center of the square. The reflection of the line over a diagonal of the square is also drawn, dividing the square into four regions. Given that the ratio of the area of the smallest region to the largest region is $20 : 23$, then the area of the smallest region can be expressed as $\frac{p}{q}$, in simplest terms. Find $p + q$.

27. If $EEL \times ED = 2023$ and E, L, D are different digits, what is $E + L + D$?

28. In a class of 100 students, 30% raise their left hand, 25% raise their right hand, and 20% raise both hands. How many students do not raise either hand?

29. Express $0.\overline{2023} = 0.20232023...$ as a fraction $\frac{p}{q}$ in simplest terms. What is $p + q$?

30. What is the sum of the digits of $10^{2023} - 10^{2006}$?

31. Alice knows how to correctly use PEMDAS, but Bob mistakenly does the operations in reverse! In other words, he does addition and subtraction before multiplication and division. They are both trying to evaluate

$$1\_2\_3\_4,$$

where the blank spaces can be replaced by any of $+, -, \times$ or $\div$. What is the maximum possible difference between their answers for their individual choices of operations?

32. Two circles are centered around the origin, one with radius 2 and the other with radius 14. The equation for the area of a circle is πr^2 where π is a constant and r is the radius of the circle. What is the ratio of the area in the larger circle but not in the smaller to the area of the smaller circle?

33. Let N be the result obtained by multiplying together $1 \times 2 \times \ldots \times 10001$. What is the remainder when N is divided by 10002?

34. Jenny is an efficient pizza worker. What is the largest number of pieces she can cut out of a pizza with exactly 6 cuts?

35. The Fibonacci Sequence is comprised of numbers such that each number is the sum of the previous 2 numbers: $1, 1, 2, 3, 5, 8, 13, 21 \ldots$ Consider the first 15 terms of this sequence. If x is the sum of the even terms and y is the sum of the odd terms, find $x - y$.

36. A rhombus has a perimeter of 52 and a diagonal of length 10. What is the area of the rhombus?

37. Piplup created a $4 \times 4 \times 4$ wooden cube by combining sixty-four $1 \times 1 \times 1$ wooden cubes. Between any two wooden cubes sharing a face, Piplup used 1 drop of glue. How many drops of glue did Piplup use in total?

38. An ordered pair of numbers is called *good* if they are the same number of digits long and each pair of corresponding digits (those in the same position) differ by 1. For example, $(38, 47)$ and $(509, 618)$ are good, but $(19, 20)$ and $(1, 12)$ are not. How many good pairs of two-digit numbers are there?

39. Consider the square of lattice points with vertices of $(0, 0), (0, 4), (4, 4)$ and $(4, 0)$. How many isosceles triangles with all vertices on lattice points can be formed with one side on the edge from $(0, 0)$ to $(4, 0)$?

40. In some numbers, the number of times a digit shows up in the number is the value of the digit. For example, in 5255525, 2 shows up 2 times and 5 shows up 5 times. How many of these numbers exist if they must be 7 digits long?